A CASE STUDY IN UNIFORM CONDITIONING OF LOCAL RECOVERABLE RESERVES ESTIMATION FOR JELŠAVA MAGNESITE DEPOSIT – LEVEL 220

PRÍPADOVÁ ŠTÚDIA ROVNOMERNÉHO PODMIEŇOVANIA PRE ODHAD ZÁSOB MAGNEZITOVEJ RUDY NA LOŽISKU JELŠAVA – OBZOR 220

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Abstract

A practical case study for estimation of magnesite grade in Jelšava deposit in Slovakia is presented. The studied part of the deposit is a new level 220, vertically extended to the level 323, presently being exploited. Based on exploratory data analysis, the distributions of studied variables were modelled using Gaussian anamorphosis and transformed into normal distribution. The structural analysis and consequential variography was made for transformed variables in 3D space, resulted in a complex structural model of the spatial variability. Following the structural model, the average grades were simulated for the projected selective mining units (SMU) 16x16x6 m (panels). For recoverable reserves estimation purposes, the raw variables were transformed into Gaussian space for SMU volume as well as discretizating blocks 4x4x6 m inside each SMU. The results of the previous simulations and the change of support models were used for recoverable reserves estimation by uniform conditioning for series of different cut-offs.

Abstrakt

Predkladaný článok sa zaoberá nelineárnym modelovaním vyťažiteľných zásob magnezitovej rudy ložiska Jelšava. Študovanou časťou ložiska bol doposiaľ neťažený obzor 220 po vyššie ťažený obzor 323. Na základe štatistickej analýzy prieskumných dát boli v prostredí ISATISTM modelované distribúcie jednotlivých študovaných premenných a transformované do normálneho rozdelenia Gaussovou anamorfózou. Pre takto transformované premenné bola urobená štruktúrna analýza a variografia v trojrozmernom priestore. Na základe výsledného štruktúrneho modelu boli simulované priemerné obsahy jednotlivých premenných pre veľkosti projektovaných ťažobných jednotiek 16x16x6 m (SMU). Hodnoty pôvodných premenných boli transformované do Gaussovho rozdelenia pre dané SMU, ako aj pre veľkosti diskretizačných blokov 4x4x6 m obsiahnutých v každej SMU. Výsledky získané simulovaním priemerných obsahy jednotlivých premenných a Gaussovej anamorfózy pre jednotlivé veľkosti nositeľa informácie boli použité pre samotný výpočet vyťažiteľných zásob metódou rovnomerného podmieňovania na základe rozdielnych podmienok bilančnosti.

Key words: distribution modelling, Gaussian anamorphosis, variography, uniform conditioning, support

INTRODUCTION

Geostatistics, in combination with other statistical methods, provides a wide range of mathematical tools that can be used for estimations that are based on the assumption that a mineralized phenomenon at one point in space can be considered as one realization of a random process. A set of spatially distributed realisations is then considered result of a random spatial process. Nowadays geostatistics can not be ignored when dealing with the reserves evaluation. It is now of common use in a large number of companies from the exploration to the production stages [3].

A good knowledge of tonnage and grades within an orebody is essential in order to assess the economic feasibility of putting the mine, or its part, into production. An orebody is always made of several types of ore and waste minerals. Separating the ore from the waste during exploration and mining is almost impossible to achieve for many reasons: the geological boundaries between ore and waste is seldom clear in nature, and conversely, boundaries defined on economic criteria, or cut-off grades, do not even match any geological reality [4]. The real grades of SMU are unknown during the exploitation of the mining. Therefore the decision to send the material to the ore plant or to the waste plant is still taken from the estimated grade and not from true grade. As a consequence it is not possible to avoid sending blocks to the wrong destination: rich block will end up on the waste dump because they are estimated as poor and vice versa.

1 PRESENTATION OF THE AVAILABLE DATA

Studied deposit is located 3.5 km NE of Jelšava (**Fig. 1**). Carbonate body has the directional length 4.5 km, course NE – SW with inclination 55 – 60° to SE and maximal thickness 600 m. Magnesite area in the western part of the deposit (Dúbrava body) reaches a thickness of 70 – 80 m. This is proven in the inclined length 1500 m and assembled in the lower part of carbonate body. The most intensive mineralization is in the middle part – Miková [9].

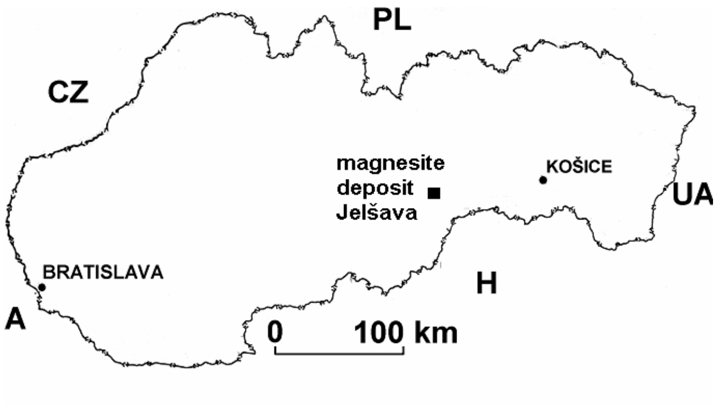

Fig. 1 Jelšava deposit's localization map.

The deposit has been extensively explored from surface deviated diamond drilling [18] and underground exploration drives, cross-cuts and drilling fans. The sample database consists of a combination of diamond drillhole samples (6214 cores), drilling fan samples (4593 cores) and channel samples (1646 samples). The final database consists of the grade analyses for content of MgO [%], CaO [%] Fe_2O_3 [%] and SiO_2 [%] together with ID, X, Y and Z coordinates. The input data file for the geostatistical software ISATIS^{TM} respected the line structure of the drillholes, drives and cross-cuts. The situation with available data is shown in **Fig. 2**.

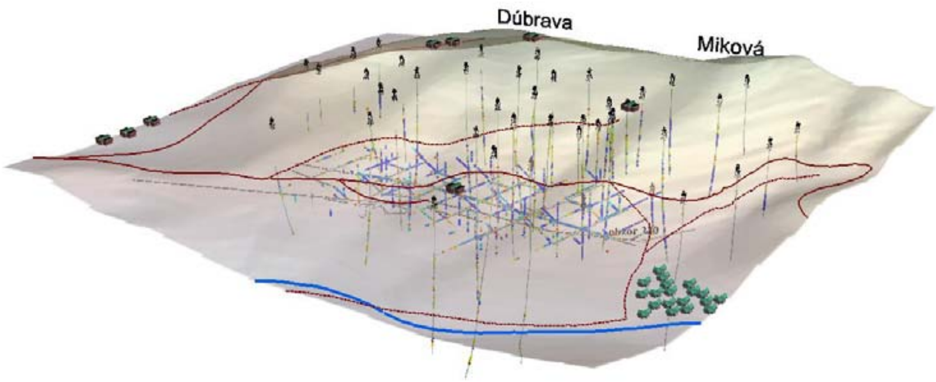

Fig. 2 3D visualization of available data.

2 EXPLORATORY DATA ANALYSIS

2.1 Statistical analysis

Because of the combination of different sample types, the first task of the exploratory data analysis consisted in testing of data compatibility by F and t tests. Statistical analysis confirmed that the grade analyses of these different sample types from different sampling campaigns come from the same parent population. **Table 1** shows some basic descriptive statistics.

Tab. 1 Descriptive statistic of variables studied.

variable	number	min [%]	max [%]	mean [%]	standard deviation [%]	variance $[\%]^2$	skewness	kurtosis
SiO_2 [%]	11964	0.01	76.80	1.38	3.31	10.95	7.9	98.93
Fe_2O_3 [%]	9766	0.24	9.97	3.54	0.78	0.62	-0.15	2.93
CaO [%]	11964	0.08	43.05	11.03	10.13	102.66	0.74	-0.99
MgO [%]	11964	4.08	50.02	35.28	8.73	76.15	-0.72	-0.92

2.2 Regularization

One prerequisite of data analysis is that the samples all represent an equal volume. This is called the support of the sample [8]. It is very important in estimation to work with equal support (volume) samples. The regularization is an essential phase of a study using 3D data and especially in the mining industry [13]. The idea is that all the bases of geostatistics will consider each datum of the same importance, prior to assigning a weight in the kriging process. This does not make sense if all the data do not represent the same amount of material. The accepted way of ensuring equal support of all samples within domain being estimated is to compose the samples into equal lengths [2]. The available data were measured on different support size by means of lengths of samples, ranking from 0.3 m to 59 m, due to the preferential sampling of the high grade magnesite in carbonate. Therefore it was necessary to create a replica of the initial data set where all the variables of interest in the input file have been converted into composites of equal lengths. Because of line structure of the data set we used the regularization on constant length 6 m which is the bench length for ascending horizontal slicing method of mining. The minimal allowable length was 3 m, approximately equal to the average length of original samples. The regularization resulted in avoiding the bias of preferential sampling of high grade zones. The following **Fig. 3** shows experimental histograms of MgO grade before regularization and after it. From these two histograms it is possible to see clearly the better representation of the statistical behaviour of the MgO contents where on the histogram before regularization there is underestimated population of the low values of the MgO contents. Conversely, histogram after regularization clearly shows two populations of the MgO contents - the low value population of the MgO contents in dolomitic magnesite and the high value population of the MgO contents in magnesite itself.

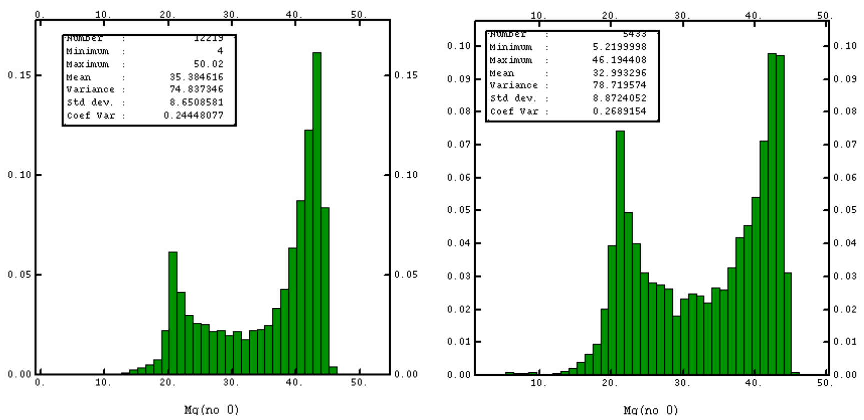

Fig. 3 Experimental histogram of MgO content before regularization (on the left) and after it (on the right).

2.3 Correlation analysis

Except for the high negative correlation between MgO and CaO (-0.97) there is also apparent correlation between these two variables and Fe_2O_3 variable (**Fig. 4**) – positive one for MgO/Fe_2O_3 (0.66) and negative for CaO/ Fe_2O_3 (-0.72). Because this variable is undersampled, those relationships were used for multivariate approach in recoverable reserves modelling.

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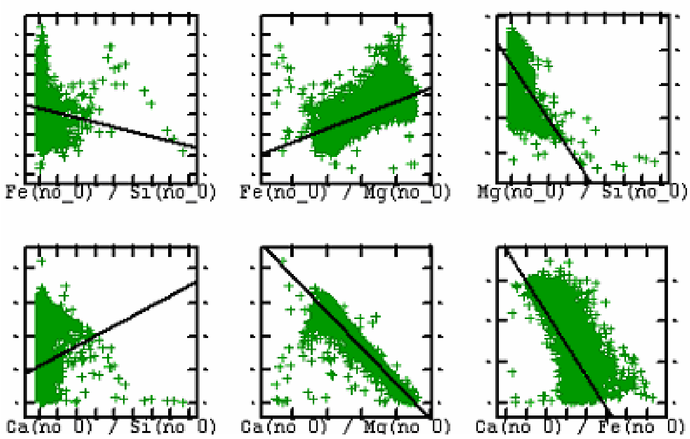

Fig. 4 Correlation analysis of the studied variables.

3 GEOSTATISTICAL MODELLING OF RECOVERABLE RESERVES

3.1 Why non-linear approach?

Estimation of recoverable reserves calls for the aspects of the mining project. The most important one is the volume on which the decision between ore and waste is done. This aspect is called **support effect**. Any mining engineer knows that recovered grades are lower when selectivity is poor, in other words, the bigger mining units, the lower grades. The average grade of a huge block of thousands of cubic meters can be considered average of the grades of smaller blocks of a few cubic meters contained in the big block. The distribution of grades of huge blocks is obviously less scattered than that of the smaller ones or compare to the samples. The only grades which are known experimentally are those of the samples. In order to predict the distribution of grade for blocks of different dimensions geostatistics provides models of change of support that are based on the experimental histograms of sample grades as well as their spatial correlation through the variogram [3].

The SMU's volume for early development of level 220 was given by dimensions 16x16x6 m. Recoverable reserves were estimated for this volume and for series of different cut-offs given in **Table 2**. The ore tonnage and metal quantity for each SMU afterwards panel) was obtained on the basis of the blocks 4x4x6 m within the panel, according to the exploitation progress.

Tab. 2 Series of given cut-off values.

Cut-off	MgO [%]	CaO [%]	SiO_2 [%]	Fe_2O_3 [%]
1	> 42.5	< 1.2	< 0.3	< 3.5
2	> 41	> 1.2	> 0.3	> 3.5
3	> 39.5	> 3.2	> 0.7	> 3.7
4	> 38	> 5.2	> 1.1	> 3.9
5	> 36.5	> 7.2	> 1.5	> 4.1
6	> 35	> 9.2	> 1.9	> 4.3
7	< 35	> 11.2	> 2.3	> 4.5

3.2 Gaussian point anamorphosis modelling

Uniform conditioning method considers the average grade of the panels as "known", and then the distribution of blocks within each panel is calculated directly by using the anamorphosis function to take into account the change of support. Because uniform conditioning depends heavily upon the quality of given "reality", it must be estimated. That estimation can be performed by any interplation method for instance using ordinary kriging [20]. Due to the complex (skewed, multimodal, …) experimental distributions of studied variables, the better way is to simulate the average grade. Geostatistical simulation techniques generate realizations with the normal, Gaussian, distribution. Thus it is necessary to perform the simulations in the Gaussian space and apply at the end a back-transformation using the anamorphosis function ϕ [7]. The objective of simulating the block grades is achievable by averaging point grades that make a discretization of the blocks. It is compulsory to perform that averaging on the point grades with the raw distribution. Doing the other way (average of normal values then back-transformation) is not correct for the simple reason that the average of Gaussian values is no more a normal variable [21]. Hence, the anamorphosis function that is required is the anamorphosis on the sample (point) support. The transformation of the data is achieved by fitting a Hermite polynomial function to the experimental distribution linking raw (*Z*) and Gaussian (*Y*) values [17]:

$$Z(x) = \phi[Y(x)] = \sum_{n=0}^{N} \frac{\psi_n}{n!} H_n[Y(x)], \qquad (1)$$

with *n* polynomial coefficients $\psi_n/n!$ for *N*-th order of Hermite polynomial H_n.

During modelling of the anamorphosis function we are looking for [6]:

- minimal differences between basic statistics of raw and back-transformed variable,
- the mean of the transformed variable is 0 and its variance is equal to 1 as close as possible,
- the average of differences between raw and back-transformed values is 0 and variance minimal,
- the shapes of the experimental histograms of raw and back-transformed variables is the same,
- correlation between raw and back-transformed variables is maximal positive,
- differences between grade/tonnage curves of raw data and back-transformed ones are minimal.

Fig. 5 shows an example of transformation of the MgO values onto normal (Gaussian) variable and back-transformation obtained by using of 37 Hermite polynomials.

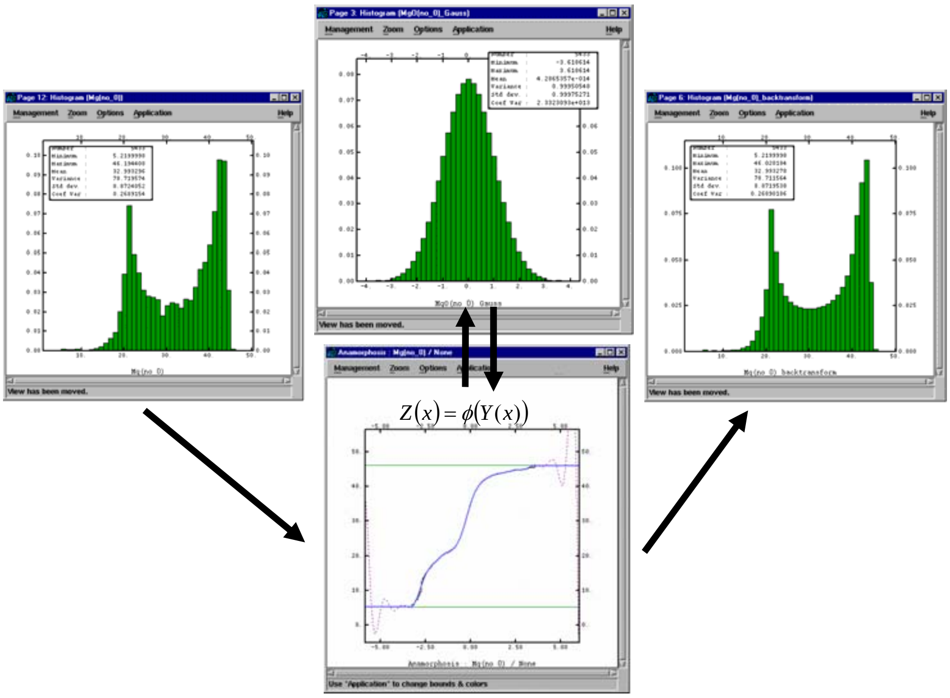

Fig. 5 Point anamorphosis modelling and variable transformation for MgO values (left – histogram of raw data, middle – data transformation, right – back-transformation).

3.3 Art of variography

The reference plane for calculation of directional experimental variograms was the vertical one with azimuth of 7° (N07E), covering drilling fans, diamond drillholes and samples from the cross-cuts. Perpendicular plane covered samples from the drives. The experimental variograms were calculated along lines for three directions: vertical, N07E and E07S. Lag distance was 60 m, number of lags 13 with lag tolerance of 0.5 (or 50 %).

Experimental variograms were calculated for both, variables transformed into Gaussian space for simulation purposes, and for raw variables, used later for Gaussian anamorphosis modelling with block support correction.

Mathematical models of basic variogram structures were fitted on calculated experimental variograms [12]. Because of evident undersampling of $Fe2O_3$ values and their relation to MgO and CaO values, these three variables were modelled together, resulted in the final model of coregionalization [5] (**Fig. 6**). Variable SiO_2 was modelled separately. The quality of the fits achieved was checked by cross-validation procedure [1]. Search plan for cross-validation, and later for simulations and estimations, was optimized by the moving neighbourhood parameters:

- 3D axis of search ellipse and search directions,
- number of angular sectors for restriction on the number of samples from one drillhole,
- minimum and optimum number of values per sector,
- maximum number of consecutive empty sectors to avoid extrapolation,
- minimum distance between two samples,
- maximum distance without any sample.

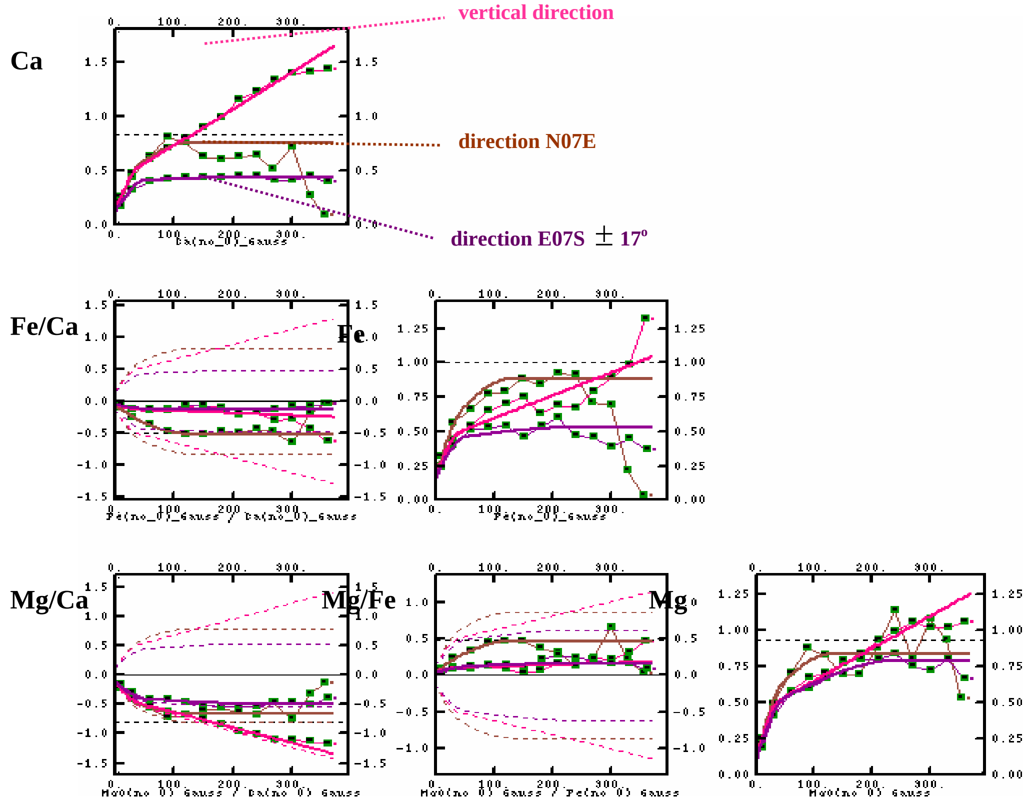

Fig. 6 Experimental variograms and final model of coregionalization for MgO, CaO and Fe_2O_3.

Final model represents the following features described by five basic variogram structures that have been fitted:

1. nugget effect represents presence of a structure smaller than minimal sampling distance,
2. spherical model with range 40 m and anisotropy coefficients (0.9, 1, 0.8) along N07E, vertical and E07S directions representing geometrical anisotropy for short distances [1, 2, 11, 12, 13, 20],
3. spherical model with range of influence 250 m along E07S direction not defined along others directions, representing the bearing of trends of magnesite lentils in carbonates [2, 11, 13, 20],
4. spherical structure with range of influence 130 m along N07E representing zonal anisotropy between this direction and perpendicular one,
5. 1st generalized covariance structure along vertical direction representing trend in dip of magnesite lentils.

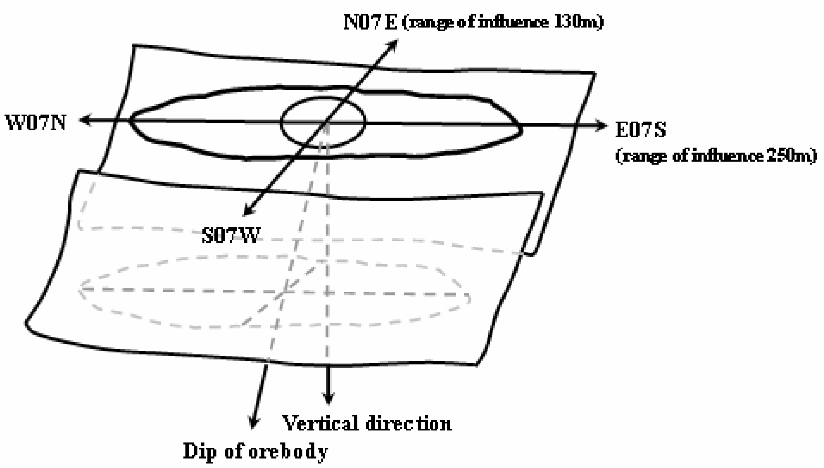

Fig. 7 The schematic sketch of the variography results.

3.4 Conditional simulation of transformed variables

Geostatistical simulation is a process of generating of one realization from all possible realizations in given point or volume [15]. That simulation must reproduce statistical and geostatistical characteristics of variability (histogram and variogram) of studied variable. That means that geostatistical simulation generates a set of values forming one of infinite possible realizations with following criterions [14]:

- in experimental value locations are simulated values the same as data,
- simulated values have the same model of variogram as the experimental ones,
- simulated values follow the same distribution as the experimental ones.

A conditional simulation is therefore a realization randomly selected from the subset of realizations that match the sample point values. Equivalently, it is a realization of a random function with a conditional spatial distribution [11].

Turning bands [7, 11, 14] were used to obtain 100 realizations of the average values for panels 16x16x6 m with discretization 7x7x2 points. The simulations were conditioned by sample values by kriging with the same moving neighbourhood as that used for cross-validation. **Fig 8** shows the final mean simulation of average values of MgO contents for panels 16x16x6 m, back-transformed from Gaussian space using the anamorphosis modelled previously.

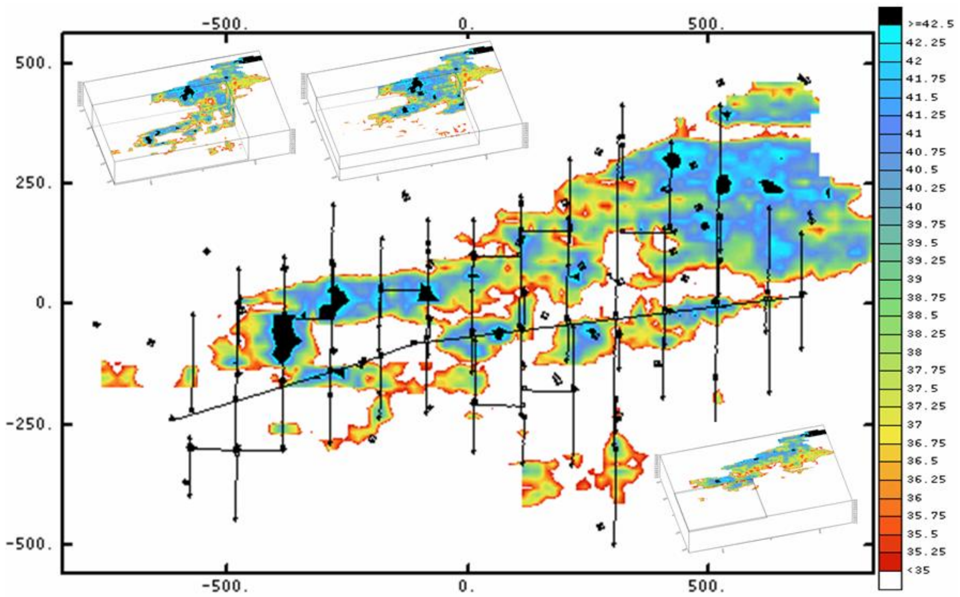

Fig. 8 Simulation of MgO the average grade for panels 16x16x6 m.

3.5 Gaussian anamorphosis modelling for blocks and panels

To achieve the uniform conditioning we have to provide:

- "the reality" of the average panel grades,
- two anamorphosis functions, one for block and one for panel support, i.e. 16x16x6 m and 4x4x6 m respectively.

The first step was made in previous part by geostatistical simulation. Because we've already simulated the reality, we can directly calculate using anamorphosis function the tonnage *T* and metal *Q* of blocks located randomly inside the panel, conditionally to the Gaussian value of the panel. The idea is that the blocks of a small volume comprise the proportion of panel's volume according to a given cut-off [16].

Application of the anamorphosis for different support (*x* for samples, *v* for blocks and *V* for panels) we get the following relationships:

$$Z(x) = \phi(Y(x)) \quad \Rightarrow \quad Z(v) = \phi_r(Y_v) \quad \Rightarrow \quad Z(V) = \phi_R(Y_V). \qquad (2)$$

For the discrete Gaussian model, the point values have an anamorphosis function given in (1). For the block values *v* have a different one ϕ_r:

$$Z(v) = \phi[Y(v)] = \sum_{n=0}^{N} \frac{\psi_n}{n!} r^n H_n[Y(v)], \qquad (3)$$

where *r* is the correlation between $Y(v)$ and $Y(x')$ with value at *x´* chosen at random in *v*. It is determined from the variance of *Z*(*v*):

$$\operatorname{var} Z(v) = \sum_{n=1}^{N} \frac{\psi_n^2}{n!} r^{2n} = \operatorname{var} Z(x) - \bar{\gamma}(v, v'), \qquad (4)$$

where $\operatorname{var} Z(x) = \sum_{n=1}^{N} \psi_n^2 / n! = \sigma^2$ and $\bar{\gamma}(v, v')$ is an average variogram between each pair within volume *v* calculated using a discretization of the block.

Similarly we have this model for panel *V*:

$$Z(V) = \phi[Y(V)] = \sum_{n=0}^{N} \frac{\psi_n}{n!} R^n H_n[Y(V)], \qquad (5)$$

where the correlation *R* between $Y(V)$ and $Y(x')$ is obtained from:

$$\operatorname{var} Z(V) = \sum_{n=1}^{N} \frac{\psi_n^2}{n!} R^{2n}. \qquad (6)$$

Multiplying the point coefficient ψ_n by R^n is the same as multiplying the block coefficient $\psi_n.r^n$ by $(R/r)^n = r_{vV}^n$. The coefficient r_{vV}^n, which is less than 1, is correlation between $Y(V)$ and $Y(v')$ for random *v´* inside *V* [16].

Fig. 9 shows the anamorphosis function modelling for panels 16x16x6 m and blocks 4x4x6 m for MgO, based on the raw variogram models showing the same features as the transformed ones. We can clearly see decreasing of variability going from samples (blue) to blocks (green) and panels (red).

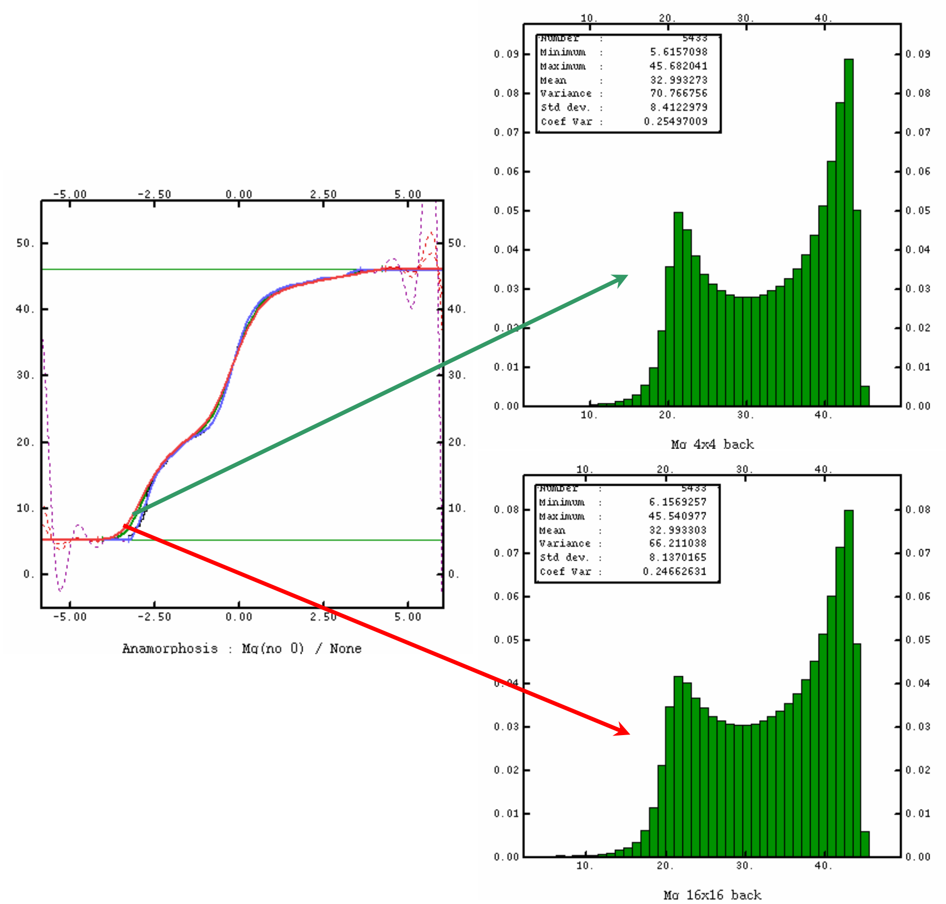

Fig. 9 Modelling of Gaussian anamorphosis for blocks (up) and panels (down) – MgO.

3.6 Uniform conditioning

Linear estimation methods such as ordinary or simple kriging commonly fail to provide unbiased estimates of recovered ore and metal tonnage after cut-off which means that a mining project can be exposed to undue risk. This risk is significant when the selective mining units are small with respect to the data spacing which results in too smoothed estimates. Non-linear estimation techniques, as uniform conditioning or disjunctive kriging, are then necessary for estimation of local grade/tonnage curves [17]. Non-linear estimation techniques open up the possibilities for the calculations of unbiased estimations of ore tonnage and metal quantity for different cut-offs and supports. Uniform conditioning method was chosen from the family of available non-linear techniques. The details on selection of method for the presented project are given in [19].

From previous part we know how to get block or panel values using anamorphosis models. Considering a block *v´* randomly located in panel *V* we have:

$$Z(v') = \sum_{n=0}^{N} \frac{\Psi_n}{n!} r^n H_n[Y(v')] \qquad (7)$$

So we change from block to panel and we can obtain:

$$E[Z(v')|Z(V)] = \sum_{n=0}^{N} \frac{\Psi_i}{n!} (R)^i E[H_n(Y(v'))|Y(V)] = \sum_{n=0}^{N} \frac{\Psi_i}{n!} (R)^i r_{vV}^i H_i Y(V) = Z(V). \qquad (8)$$

In other words, if we know the panel grade *Z*(*V*) we also know the mean of the block grade inside the panel *V* because it is exactly the same value. That means if we know the panel grade we can deduce the estimate of any function of *Z*(*v´*), and hence of *Y*(*v´*), from it. If we assume that the bivariate distribution of *Y*(*V*)|*Y*(*v´*) is normal, then we know that given that *Y*(*V*) = *y*(*V*), the variable *Y*(*v´*) has a normal distribution with mean $r_{vV}\, y(V)$ and variance $1 - r_{vV}^2$ [16]. The ore tonnage at a given cut-off $z_c = \phi_v(y_c)$ is:

$$T(z_c) = E\left[I_{Z(v')\geq z_c} \middle| Z(V)\right] = E\left[I_{Y(v')\geq y_c} \middle| Y(V)\right] = 1 - G\left\{\frac{y_c - r_{vV}Y(V)}{\sqrt{1-r_{vV}^2}}\right\}. \tag{9}$$

The corresponding quantity of metal is:

$$\begin{aligned} Q(z_C) &= E\left[Z(v')I_{Z(v')\geq z_c} \middle| Z(V)\right] = E\left[\phi_v[Y(v')]I_{Y(v')\geq y_c} \middle| Y(V)\right] \\ &= \sum_{i=0}^{N} r_{vV}^i H_i[Y(V)] \sum_{j=0}^{N} \psi_j r^i \int_{y_c}^{+\infty} H_j(y) H_i(y) g(y) dy. \end{aligned} \tag{10}$$

Finally, the average recoverable grade $M(z_c)$ equals the metal quantity divided by the ore tonnage:

$$M(z_c) = \frac{Q(z_c)}{T(z_c)}. \tag{11}$$

CONCLUSION

This paper presents using of non-linear geostatistical approach for recoverable reserves estimation by uniform conditioning in ISATIS™ software. Regardless of long time existence of non-linear geostatistical methods, this was the first time using in Slovakia as a practical study. The reason for that study was the demand for a complex numerical model of recoverable reserves from the mining company. Necessity of that model comes from changing the exploitation method from the chamber-pillar to the slicing bench with emphasis on selectivity and recoverability for more effective exploitation of the deposit. For this reason, the final grade/tonnage curves can't be published here. The results presented in this case study, and summarized in **Fig. 10**, were implemented into GIS environment [10] created especially for the mining company, allowing real-time three dimensional querying and rotation of orebody. It resulted in quicker and easier comparison of the resource model to the real exploitation, leading to more frequent resource updates and following scheduling and prediction.

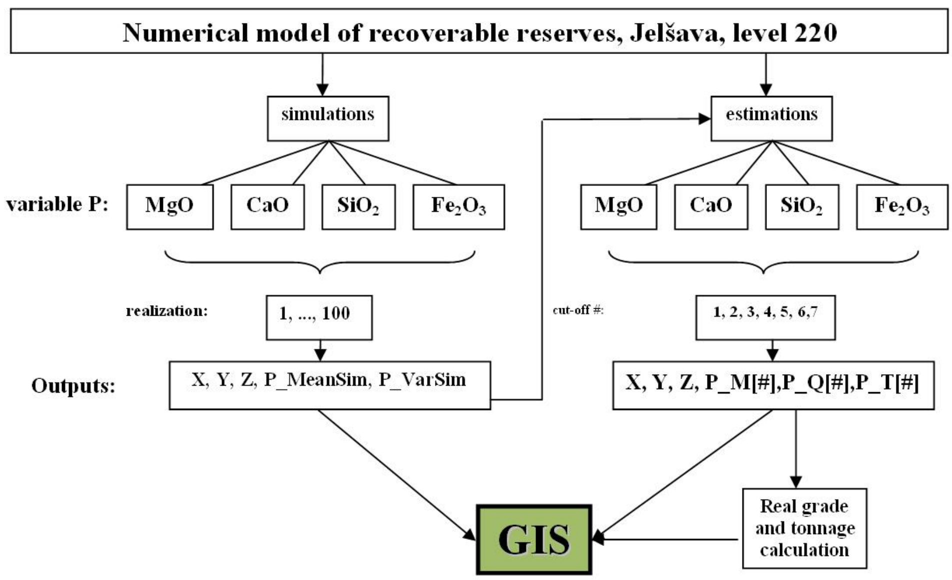

Fig. 10 Results of non-linear estimation of recoverable reserves for Jelšava – level 220.

Acknowledgment

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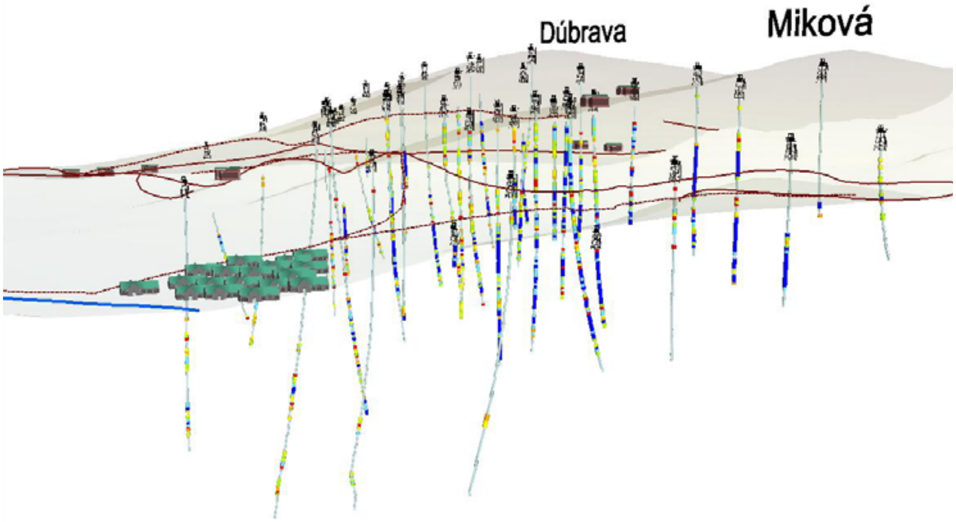

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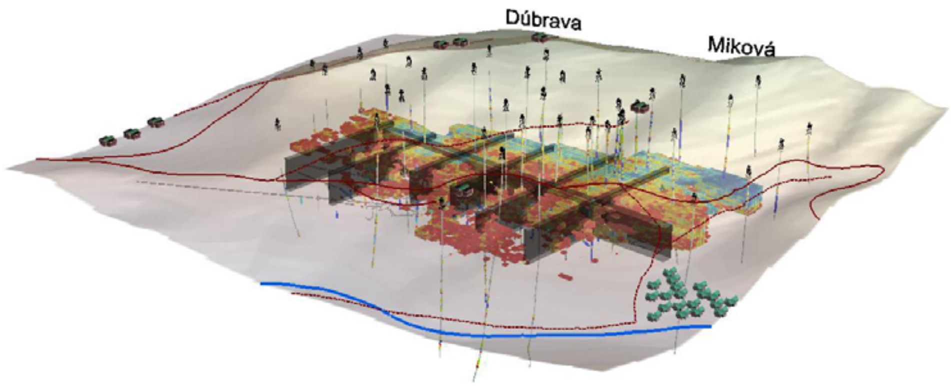

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RESUMÉ

Predkladaný článok sa zaoberá teoretickým pozadím a aplikáciou nelineárneho prístupu rovnomerného podmieňovania k odhadom vyťažiteľných zásob rudy na novo otvorenom obzore 220 magnezitového ložiska Jelšava. Odhad vyťažiteľných zásob, buď globálnych alebo lokálnych, sa stáva štandardnou geoštatistickou aplikáciou v banskom priemysle, ale aj iných odvetviach ľudskej činnosti. Z toho vyplýva, že geoštatistika ako taká nemôže byť ignorovaná v procese odhadu a oceňovania zásob na ložisku. Geoštatistické prístupy a metódy sú dnes bežne používané mnohými ťažobnými spoločnosťami, a to od etapy prieskumu až po produkciu. Avšak čas kedy boli dostupné len variogramy a lineárne krigovanie je dávno preč. Prax si vyžaduje viac vo vzťahu k odhadom zásob na základe rôznych bilančných podmienok (cut-off).

V geoštatistickom zmysle, znamenajú vyťažiteľné zásoby tonáž rudy T_V a množstvo kovu Q_V obsiahnutého v tejto tonáži pre danú ťažobnú jednotku *V*. Priemer vyťažiteľného obsahu chemizmu *M* je potom výsledkom podielu množstva kovu a tonáže rudy. V praxi nie je zaujímavé pracovať s absolútnou tonážou, ale len tonážou tých ťažobných jednotiek, ktorých obsah chemizmu Z_V je vyšší ako určitá bilančná, cut-off, podmienka ťažby z_c. Množstvo kovu je potom počítaný ako táto tonáž vynásobená obsahom chemizmu ťažobnej jednotky. Tento typ formalizmu je používaný k vyjadreniu dvoch premenných vo výraze indikátorov:

$$T_V = I_{Z_V \geq z_c} = \begin{cases} 1 & \text{ak} \quad Z_V \geq z_c \\ 0 & \text{ak} \quad Z_V < z_c \end{cases}, \quad \text{a} \quad Q_V = Z_V I_{Z_V \geq z_c} = \begin{cases} 1 & \text{ak} \quad Z_V \geq z_c \\ 0 & \text{ak} \quad Z_V < z_c \end{cases}.$$

Pre výpočet absolútnej tonáže a množstva kovu sú T_V a Q_V vynásobené objemom ťažobnej jednotky a objemovou hustotou ťaženej rudy. Avšak vyjadrenia vyššie predpokladajú, že v čase ťažby je skutočná hodnota obsahu chemizmu ťažobnej jednotky známa. To však nie je pravda. Počas produkcie sú známe iba odhady tohto obsahu chemizmu Z_V^* a tie sú odhadnuté na základe dostupných vzoriek, čo vedie k chybám v klasifikácií medzi rudou a jalovinou. Takže zásoby v tomto vyjadrení sú v skutočnosti "ideálnymi" zásobami a pre odhad skutočných zásob je potrebné počítať s informačným efektom, ktorý je možné odvodiť na základe modelov distribúcií skutočných ťažobných jednotiek a ich odhadov, ako aj ich spoločnej a kondičnej distribúcie. Každý banský inžinier vie, že čím je väčšia ťažobná jednotka, tým je v priemere nižší obsah chemizmu. Priemerný obsah chemizmu ťažobnej komory s niekoľkými tisíc kubických metrov môže byť vnímaný ako priemer obsahov chemizmu malých mikroblokov s obsahom niekoľko kubických metrov obsiahnutých v danej komore. Distribúcia obsahu chemizmu komory je zvyčajne menej rozptýlená ako malých mikroblokov alebo dostupných vzoriek. Jediný obsah chemizmu, ktorý je známy je obsah chemizmu vo vzorkách. Aby bolo možné predpovedať a odhadnúť distribúciu chemizmu pre ťažobné jednotky rôznych rozmerov, poskytuje geoštatistika modely zmeny nositeľa veľkosti informácie (support), založené na experimentálnom histograme obsahu chemizmu vzoriek, ako aj ich priestorovej korelácií v podobe variogramu.

Napriek dlhodobej existencií nelineárnych geoštatistických techník, a ich použitiu v rôznych odvetviach, počnúc ťažbou nerastných surovín, prípadová štúdia popísaná v tomto článku je „prvou lastovičkou“ na území Slovenska. Dôvodom pre výber tohto prístupu odhadu vyťažiteľných zásob magnezitovej suroviny, v zmysle odhadu tonáže, množstva kovu a priemerných obsahov sledovaných zložiek rudy pre rôzne bilančné podmienky (cut-off), bola požiadavka pre vytvorenie komplexného numerického modelu novo otváranej časti ložiska pre banskú prevádzku. Potreba takéhoto modelu vyplynula zo zmeny dobývacej metódy na tomto ložisku, z metódy dobývania komora-pilier na výstupkové dobývanie, s dôrazom na selektívnu ťažbu magnezitovej rudy a vyťažiteľnosť úžitkových zložiek, pre potreby efektívnejšieho využívania nerastnej suroviny ložiska. Z uvedeného dôvodu nie sú v tomto článku prezentované výsledné krivky kovnatosť/tonáž. Vyťažiteľné zásoby boli počítané pre podmienky postupu ťažby pre ťažobné jednotky o rozmeroch 16x16 m pre výšku výstupku 6m na základe ktorých je vykonávaná selekcia medzi rudou a jalovinou. Použitá metóda rovnomerného podmieňovania predpokladá že priemerné hodnoty študovaných premenných sú pre vybranú selektívnu jednotku známe. Z uvedeného dôvodu boli priemerné hodnoty jednotlivých premenných pre jednotky 16x16x6 m, transformovaných do normálneho, Gaussovho rozdelenia bodovou Gaussovou anamorfózou (**Obr. 5**), simulované metódou otáčania pásiem (*turning bands*), na základe variografie (**Obr. 6**). Pre získanie priemerných hodnôt jednotiek 16x16x6 m boli tieto diskretizované 7x7x2 bodmi. Výsledné simulácie boli spätne transformované do pôvodného priestoru rovnakou bodovou anamorfózou (**Obr. 8**). Výsledné vyťažiteľné zásoby podľa bilančných podmienok uvedených v **Tab. 2** boli odhadnuté na základe výsledkov variografie pôvodných, netransformovaných hodnôt a modelov anamorfózy pre veľkosti ťažobných jednotiek 16x16x6 m (panelov) a blokov 4x4x6 m umiestnených vo vnútri selektívnych jednotiek (**Obr. 9**)

Výsledky prezentované v tomto článku, zhrnuté v schéme na **Obr. 10**, boli zavedené do GIS systému, vytvoreného špeciálne pre potreby banského podniku. Tento GIS umožňuje rýchle porovnávanie vytvoreného modelu s reálnou ťažbou, čo vedie k efektívnejšiemu plánovaniu ťažby.