ANALYSIS OF GEODETIC NETWORK ESTABLISHED INSIDE THE DOBŠINSKÁ ICE CAVE SPACE

ANALÝZA GEODETICKEJ SIETE ZRIADENEJ V PRIESTOROCH DOBŠINSKEJ ĽADOVEJ JASKYNE

Juraj GAŠINEC [1], *Silvia GAŠINCOVÁ* [2], *Vladislava ZELIZŇAKOVÁ* [3], *Jana PALKOVÁ* [4]
Žofia KUZEVIČOVÁ[5]

[1] *Assoc. prof., Ing., PhD., Institute of Geodesy, Cartography and Geographic Information Systems, Faculty of Mining, Ecology, Process Control and Geotechnologies, Technical University of Košice, Park Komenského 19, 043 84 Košice, Slovak Republic, +421 55 602 2846*
e-mail: juraj.gasinec@tuke.sk

[2] *Assoc. prof., Ing., PhD., Institute of Geodesy, Cartography and Geographic Information Systems, Faculty of Mining, Ecology, Process Control and Geotechnologies, Technical University of Košice, Park Komenského 19, 043 84 Košice, Slovak Republic, +421 55 602 2846*
e-mail: silvia.gasincova@tuke.sk

[3] *Ing. Vladislava Zelizňáková, Institute of Geodesy, Cartography and Geographic Information Systems, Faculty of Mining, Ecology, Process Control and Geotechnologies, Technical University of Košice, Park Komenského 19, 043 84 Košice, Slovak Republic, +421 55 602 2449*
e-mail: vladislava.zeliznakova@tuke.sk

[4] *Ing. Jana Palková, Institute of Geodesy, Cartography and Geographic Information Systems, Faculty of Mining, Ecology, Process Control and Geotechnologies, Technical University of Košice, Park Komenského 19, 043 84 Košice, Slovak Republic, +421 55 602 2449*
e-mail: jana.palkova@tuke.sk

[5] *Assoc. prof., Ing., PhD., Institute of Geodesy, Cartography and Geographic Information Systems, Faculty of Mining, Ecology, Process Control and Geotechnologies, Technical University of Košice Park Komenského 19, 043 84 Košice, Slovak Republic, +421 55 602 2916*
e-mail: zofia.kuzevicova@tuke.sk

Abstract

The present article summarizes the progress and results of geodetic works during the construction of a geodetic network inside the Dobšinská Ice Cave underground space to monitor temporal and spatial changes in its ice filling. In order to objectively evaluate the changes, parameter estimations of the first- and second-order of the geodetic network from the set of field geodetic measurements were provided, and a robust analysis of the network was applied in terms of the assessment of impacts of potential outlier measurements on the network geometry.

Abstract

Predložený príspevok sumarizuje priebeh a výsledky geodetických prác počas budovania polohovej geodetickej siete založenej v podzemných priestoroch Dobšinskej ľadovej jaskyne za účelom monitorovania časových a priestorových zmien jej ľadovej výplne. V snahe objektívneho vyhodnotenia týchto zmien boli zo súborov terénnych geodetických meraní stanovené odhady I. a II. rádu geodetickej siete a z hľadiska posúdenia vplyvu potenciálnych odľahlých meraní na geometriu siete bola aplikovaná robustná analýza tejto siete.

Key words: positional geodetic network, robust analysis, Dobšinská Ice Cave

1 INTRODUCTION

The Dobšinská Ice Cave ranks among the most important world's caves. Its magnificent ice filling has remained the same for thousands of years at an altitude of only 920 to 950 meters. The ice filling is not static; it changes depending on climatic conditions and gravitational distortions. On the surface of the ice filling and on the walls of tunnels along a scenic route cut into the ice, various large and small morphological shapes, created by running and dripping water as well as air flow and sublimation of ice, are produced. In addition to a geoscience and environmental point of view, this issue is particularly important namely in terms of safety and maintaining footpaths of the scenic route for visitors. In 2010 and 2011, the mentioned issue started to be addressed in an innovative way by the project VEGA No 1/0786/10 based on the cooperation of the Institute of Geodesy, Cartography and Geographic Information Systems at the BERG Faculty, Technical University in Košice with the State Nature Conservancy of the Slovak Republic and the Slovak Caves Administration in Liptovský Mikuláš. Its main objective was the exact recording and digital modelling of changes in ice filling in caves by means of contactless measuring methods, due to their protection and operation needs. From the perspective of the objective assessment of glaciation development, it is, of course, very important to measure changes in ice filling at the highest level of precision which would be impossible without any well-built horizontal and vertical networks.

2 DOBŠINSKÁ ICE CAVE

The Dobšinská Ice Cave is situated on the southwestern border of the Slovak Paradise National Park in the Spiš-Gemer Karst which ranks among the most important karst areas in Central Europe. The cave is located in the cadastral area of the town of Dobšiná, Rožňava district, at a distance of about 18 km from the town. The cave and its surroundings belong to the National Nature Reserve Stratená [10]. The entrance to the cave, which is on the northern slope of the Duča Hill at an altitude of 969 m, has long been known as the *"ice hole"*. It takes about 25 minutes to walk from the road up to the cave. As regards natural conditions, the Dobšinská Ice Cave is part of the genetic system Stratenská Cave, consisting of 6 separate caves: *Dobšinská Ice Cave, Duča Cave, Stratenská Cave and Dog Holes Cave, Military Cave, Green Cave and Sinter Cave* (Fig. 1). In this system, five genetic levels and two horizons, of which the fourth genetic level is the most developed and the most important, were classified. The entire system was created by two underground streams – the Tiesňava brook and the Hnilec River [1].

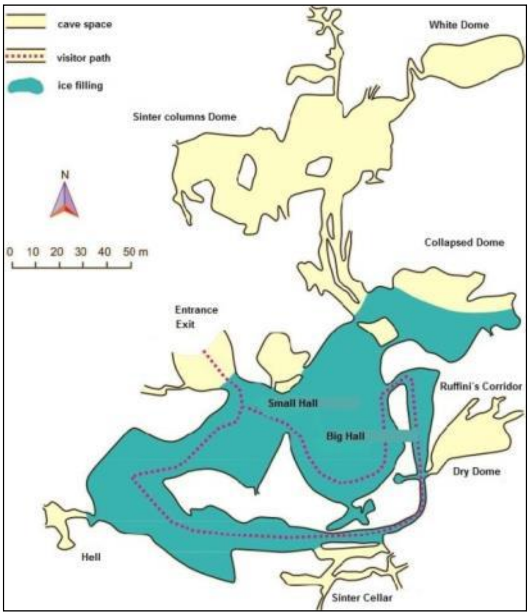

Fig. 1 Dobšinská Ice Cave [4]

Currently, the Dobšinská Ice Cave is largely filled with ice extending here and there up to the ceiling and dividing its upper part into two separate sections – the Small Hall and the Great Hall (Fig. 2).

Fig. **2** Forms of ice filling in the Small and Great Halls

3 ESTABLISHMENT OF GEODETIC NETWORK IN DOBŠINSKÁ ICE CAVE SPACES

Surveying measurements took place in collaboration with personnel of the Slovak Caves Administration based in Liptovský Mikuláš. With regard to the short, two-year duration of the VEGA project, two stages of measurements have been carried out so far. In the first stage of surveying works which took place in March 2011, spatial measurements of the Great Hall and the Small Hall of the Dobšinská Ice Cave were made by the terrestrial laser scanner Leica ScanStation C10 and through the motorized universal measuring Trimble ® VX ™ Spatial Station; in the second stage, the universal measuring station Leica Viva TS15 and the same laser scanner Leica ScanStation C10 were used. In the first stage of surveying works, positional and vertical connections to the preserved points of underground positional and vertical control was realized in the coordinate system of the Datum of Uniform Trigonometric Cadastral Network (S-JTSK) and the Baltic Vertical Datum – After Adjustment (Bpv). The vertical connection was made due to the subsequent creation of a spatial model of the Dobšinská Ice Cave. Since a considerable number of the points were damaged, it was necessary to monument a new minor geodetic control in the second phase of surveying works within solving the project. The point monumentation was performed so that any damage or destruction of the ice filling occurs, the monumented points cannot be damaged or possibly destroyed by natural processes ongoing in the cave or by visitors themselves, and good visibility of as greatest number as possible of other points in the geodetic network are preserved. The structure of the built geodetic network consisting of 11 points is demonstrated in Fig. 3.

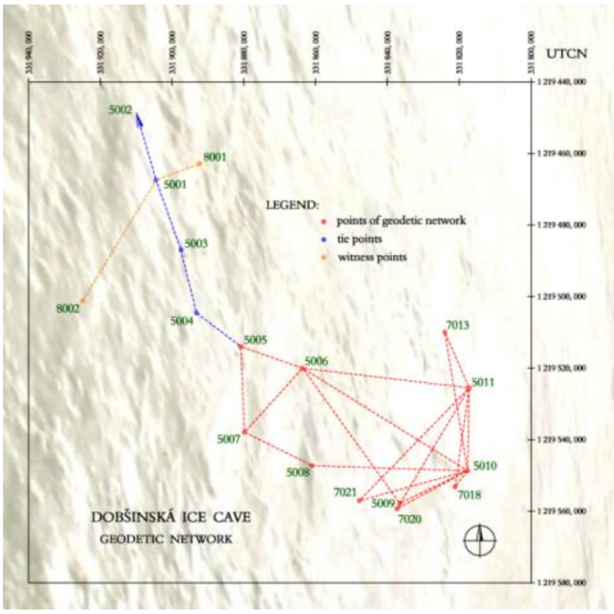

Fig. 3 The clear outline of the network of surface and underground surveying points

Surface points of the new created network were attached to the National Spatial Network; the connection was implemented via the Slovak Spatial Observation Service (SKPOS) using signals of the Global Navigation Satellite Systems (GNSS). For the static GNSS measurement lasted for three hours, two two-frequency Leica GPS1200 receivers and Leica GPS900 were used which identify the points of the orientation line 5001-5002 located approximately in a distance of 1047 m from each other. From the orientation line 5001-5002, the surface surveying witness marks No 8001 and No 8002, the points of underground control of the cave No 5004-5011 monumented in solid, unweathered parts of the rock cave ceiling with surveying pins (Fig. 4) as well as the points No 7013, 7018, 7020 and 7021 monumented with reflecting labels, were determined. With regard to the fact that the point No 5001 is monumented before entering the cave, and could be damaged e.g. during reconstruction works on the pavement, the part of the built geodetic network are also the mentioned witness marks No 8001 and 8002 which can be used in this case in subsequent geodetic measurements. The witness marks were not subject to the adjustment and the robustness analysis of the geodetic network.

Fig. 4 The monumentation of a surveying point in the Small Hall of the Dobšinská Ice Cave

The connection of the geodetic network to the S-JTSK was implemented unilaterally by the connected and oriented traverse consisting of the points 5001, 5003, 5004 and 5005 with an orientation to the point 5002 (Fig. 3) .

4 ADJUSTMENT AND ROBUSTNESS ANALYSIS OF POSITIONAL GEODETIC NETWORK

The measurement carried out in situ by using a static method was processed through the use of the Leica Geo Office software. Cartesian coordinates Y, X and Z of the measured points 5001 and 5002 in the European Terrestrial Reference System ETRS 89 (B, L, $H_{elips.}$) are the results of the processing; such as they are transformed to the coordinates Y, X and H in the S-JTSK and the Bpv systems through a transformation service provided by the Office of Geodesy, Cartography and Cadastre of the Slovak Republic (Tab. 1).

Tab. 1 The coordinates of the points 5001 and 5002 determined in the S-JTSK and Bpv systems

Point	Y [m]	X [m]	H [m]
5001	331,904.430	1,219,467.427	969.349
5002	332,194.234	1,218,472.894	871.125

The estimation of parameters of the first order of the geodetic network was implemented by the method of least squares applied to the model of adjustment of intermediary measurements with the following conditions for the unknowns:

$$\begin{aligned} \mathbf{v} &= \mathbf{A}.\mathbf{d}\hat{\mathbf{C}} - \mathbf{dl}, \\ \mathbf{0} &= \mathbf{G}^{T}.\mathbf{d}\hat{\mathbf{C}}, \\ \Sigma_{\mathbf{l}} &= \sigma_0^2.\mathbf{Q}_{\mathbf{l}}, \end{aligned} \qquad (1)$$

Where:

$\mathbf{v}$ – vector of measured quantities,

$\mathbf{A}$ – design matrix,

$\mathbf{d}\hat{\mathbf{C}}$ – vector of complements of adjusted coordinates,

$\mathbf{dl}$ – vector of complements of measured values ($\mathbf{dl}=\mathbf{l}-\mathbf{l}^0$),

$\mathbf{G}$ – datum matrix,

$\Sigma_{\mathbf{l}}$ – covariance matrix of measured values,

σ_0^2 – a priori unit weight variance factor,

$\mathbf{Q}_{\mathbf{l}}$ – cofactor matrix of measured quantities [7].

Whereas the number of measured values (n = 44, 19 measured lengths, 25 measured horizontal directions) is greater than the number of the determined parameters (k = 22, 11 points of the network for each point of the network, two coordinates are determined for each point of the network), there are redundant measurements for adjustment (LSM) in the network (Tab. 2). In order to avoid undue weighting of individual measured values due to the influence of the specific cave environment on the surveying process, the appropriate weighting coefficients of the cofactor matrix Ql (estimation of parameters of II order) were calculated by means of the method MINQUE (Minimum Norm Quadratic Unbiased Estimation) [5],[8] and are represented by an estimated standard deviation of the measured lengths $m_d = 1{,}4\ mm$ and the directions $m_\alpha = 1{,}49\ mgon$ for the motorized universal measuring station Leica Viva TS15. For testing the residues (corrections) of the measured values in terms of the detection of outlier measurements which could contaminate the set of measured values due to the influence of the cave complex physical environment on the surveying process, the Pope τ–test [3] was used:

$$T_i = \frac{|v_i|}{s_0 \cdot \sqrt{q_{v_i}}} \approx \tau_{f,(1-\alpha 2/2)}. \qquad (2)$$

Where:

T – test statistics,

$\boldsymbol{v}_i$ – corrections,

s_0 – posteriori variance factor,

q_{vi} – cofactor of corrections,

$\tau_{f,(1-\alpha 2/2)}$ – The Pope test critical value for the chosen level of significance α

Tab. 2 Coordinates and accuracy characteristics of points of the positional geodetic network

Point	Y [m]	X [m]	s_y [mm]	s_x [mm]	s_{yx} [mm]	s_P [mm]	a_s [mm]	b_s [mm]	σ [^{g}]	a_c [mm]	b_c [mm]
5005	331880.840	1219513.959	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.0	0.0
5006	331863.605	1219520.031	0.6	0.2	0.5	0.7	0.7	0.0	321.56	2.3	0.0
5007	331879.790	1219537.898	1.0	0.6	0.8	1.1	1.0	0.5	327.97	3.6	1.6
5008	331860.951	1219547.233	1.3	0.9	1.2	1.6	1.4	0.9	82.76	4.7	3.0
5009	331836.520	1219557.762	1.9	1.6	1.8	2.5	2.1	1.3	60.58	7.3	4.5
5010	331817.615	1219548.690	1.4	2.2	1.8	2.6	2.5	0.8	32.13	8.5	2.9
5011	331817.182	1219525.537	0.8	2.2	1.7	2.4	2.2	0.8	3.11	7.7	2.7
7013	331823.952	1219509.954	1.4	2.1	1.8	2.5	2.2	1.2	375.10	7.5	4.3
7018	331821.131	1219553.159	1.6	2.1	1.9	2.7	2.5	0.9	40.09	8.6	3.1
7020	331837.318	1219559.308	2.0	1.6	1.8	2.6	2.2	1.4	64.59	7.6	4.6
7021	331847.769	1219557.039	1.9	1.6	1.8	2.5	1.9	1.6	83.98	6.6	5.3

Legend:

Y, X – coordinates of points in S-JTSK,

s_y, s_x – mean coordinate errors of point in direction of axes Y and X,

s_{yx} – mean coordinate error of point,

s_P – mean point position error,

a_s, b_s – major and minor semi-axes of standard error ellipse,

a_c, b_c – major and minor semi-axes of absolute 95% confidence level error ellipse

σ – convolution of major semi-axis of error ellipse.

The error ellipses (Fig. **5**) indicate the error propagation in the direction from the fixating point 5005 to the network. The network as a whole can be characterized by the mean average coordinate error $m_{xy} = 1{,}5\ mm$ and the mean positional error $m_p = 2{,}1\ mm$.

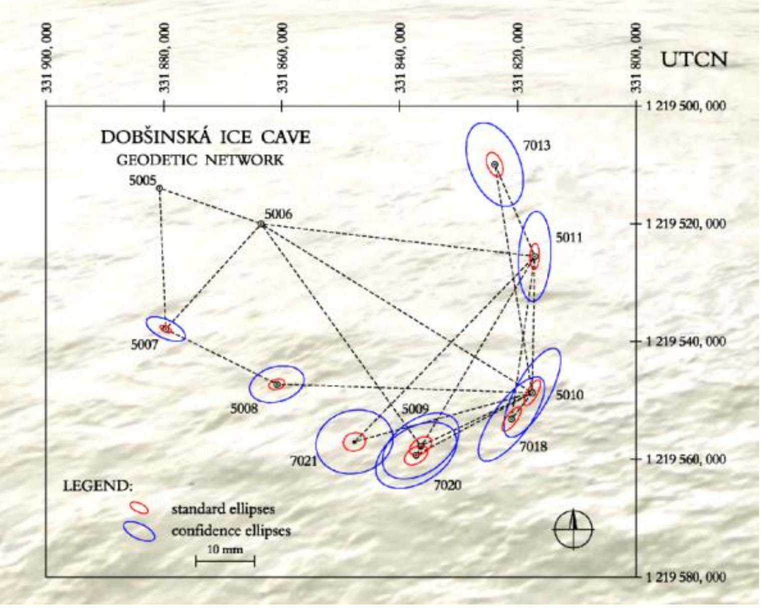

Fig. **5** Standard and confidence error ellipses

In many cases, the terrestrial measurements are tested only in a statistical sense (testing blunders in the set of measured data, testing posteriori variance factor value, testing absolute and relative confidence ellipses, testing posterior estimates of residues) [7]. In the event that a blunder in the statistical tests of estimated residues is revealed, the incorrect measurement can be corrected (in practice most excluded). A problem occurs when a blunder is not revealed during testing, or the test does not recognize any blunders. The aim of the so-called robustness analysis is to determine the degree of the network robustness – to determine the effect of undetected blunders [6]. The degree of robustness of the network is determined by its degree of deformation. The easiest way to describe the deformation of the network consists in the displacement of individual points of the network. The shifts cause a problem as their estimates are datum-dependent, i.e. their estimates depend not only on the network geometry and the accuracy of measurements, but also on the choice of the method of adjustment which has nothing to do with the deformation of the network. If the deformation is to be used to quantify the robustness of the network, then the deformation characteristics must reflect only the network geometry, the type and accuracy of measurements.

Let us denote the shift of a point P_i of the network as follows:

$$\mathbf{\Delta x_i} = \begin{bmatrix} \Delta x_i \\ \Delta y_i \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \end{bmatrix}, \tag{3}$$

in the shift vector $\mathbf{\Delta\hat{x}} = \left(\mathbf{A}^{\mathbf{T}}.\mathbf{Q}_{\mathbf{l}}^{-1}.\mathbf{A}\right)^{-1}.\mathbf{A}^{\mathbf{T}}.\mathbf{Q}_{\mathbf{l}}^{-1}.\sqrt{\lambda_0}.\frac{q_{l_i}}{\sqrt{r_i}}$ where each coordinate differences are replaced due to the simplification of notation by the symbols u_i and v_i, the symbol r_i reflects the redundancy of the network.

Then the gradient tensor in respect to the position is given by:

$$\mathbf{E_i} = grad(\mathbf{\Delta x_i}) = \begin{bmatrix} \frac{\partial u_i}{\partial x} & \frac{\partial u_i}{\partial y} \\ \frac{\partial v_i}{\partial x} & \frac{\partial v_i}{\partial y} \end{bmatrix}, \tag{4}$$

where $\mathbf{\Delta x_i}$ is the shift vector of the point P_i. The matrix **E** is a so-called matrix of deformation [2], [3] or strain matrix (of the point P_i) and is independent on the method of adjustment – datum. The strain matrix elements of each network point can be determined in several ways. The simplest of them is to obtain partial derivatives right from the shifts. Let us take, for example the point $P_i = P_0$ with the position vector $\mathbf{r_i} = (x_i, y_i) = \mathbf{r_0}$ and the adjacent points P_j with the position vectors $\mathbf{r_j}$. For the point P_i and each point P_j, two equations for two planes corresponding to the shift components u_j and v_j can be then written as follows:

$$\begin{aligned} \forall j = \ldots: \quad & a_i + \left(\frac{\partial u_i}{\partial x}\right)(x_j - x_i) + \left(\frac{\partial u_i}{\partial y}\right)(y_j - y_i) = u_j, \\ \forall j = \ldots: \quad & b_i + \left(\frac{\partial v_i}{\partial x}\right)(x_j - x_i) + \left(\frac{\partial v_i}{\partial y}\right)(y_j - y_i) = v_j, \end{aligned} \tag{5}$$

where all the partial derivatives, the absolute members a_i and b_i coordinates x_i and y_i relate to the point P_i. To determine the strain matrix of any point P_i of the network, all points of the geodetic network were used (Fig. **6**). The arrows at the connecting line of two points do not indicate the direction of measurement, but the correlation between the points.

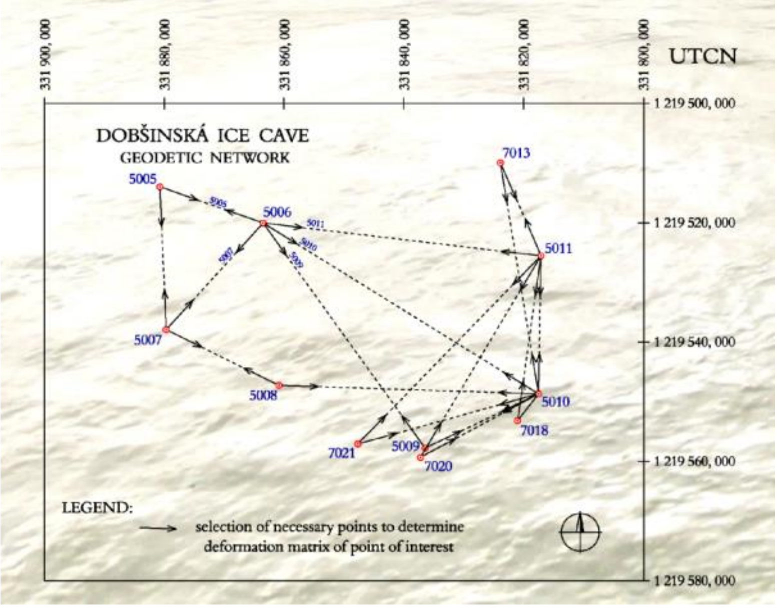

Fig. **6** Relationships between points in the analysis of robustness

Any potential change in the measurement causes a potential deformation of the entire network. To study the degree of deformation caused by potential gross errors in the measurements, only the greatest deformation of each point is to be taken into account. This greatest potential deformation corresponds to the weakest point in the network – a network can be just as strong (robust) as its weakest point.

To describe the dimension of deformation, the following is used [6]:

principal strain σ (strain in scale):

$$\sigma = \frac{1}{2} \cdot \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right), \tag{6}$$

full shear γ (strain in configuration):

$$\gamma = \frac{1}{4} \cdot \sqrt{\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2}, \tag{7}$$

local rotation ω (strain in orientation):

$$\omega = \frac{1}{2} \cdot \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right). \tag{8}$$

The values of the principal strain σ, full shear γ, and the rotation ω of each point are illustrated on the relevant maps (Fig**.** **7**, Fig**.** 8 and Fig. 9) by means of circles whose radius corresponds to the numerical value of the appropriate deformation primitive. The analysis results clearly show that the point 7018 is the weakest point in the network in terms of robustness in scale, in configuration and in orientation.

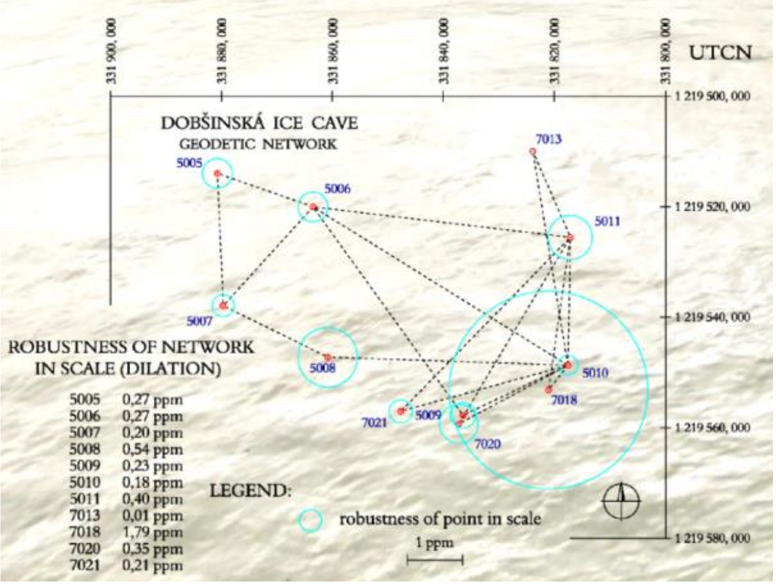

Fig. **7** Robustness of the positional geodetic network in scale

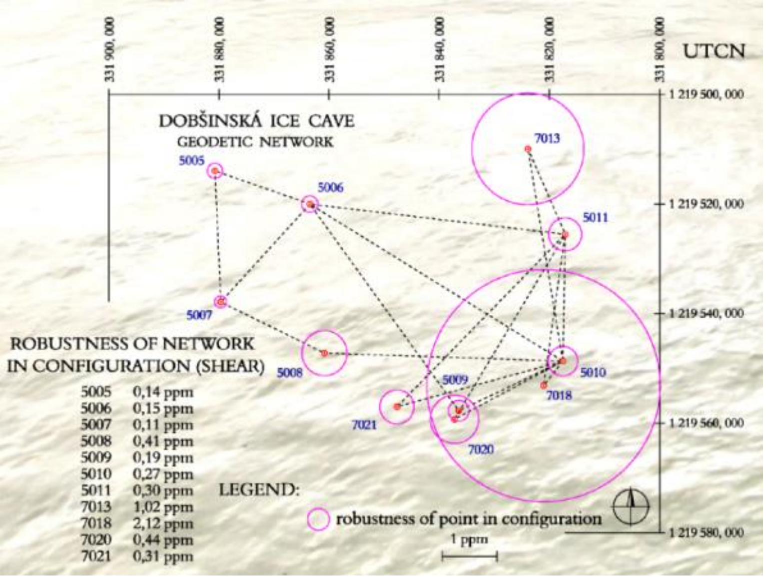

Fig. **8** Robustness of the positional geodetic network in configuration

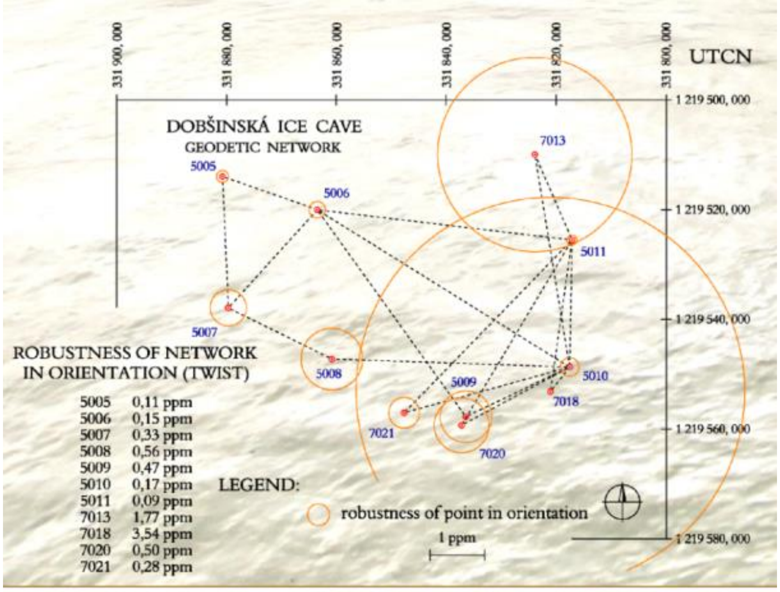

Fig. 9 Robustness of the positional geodetic network in orientation

5 CONCLUSIONS

The robustness analysis highlights the weak points in the network, the use of which may lead to biased monitoring results of ice filling. For its use, it would be appropriate to make further supporting measurements binding to the points 7013 and 7018 and effectively increase the robustness and homogeneity of the network in its weak points.

REFERENCES

[1] BELLA, P.: Glaciálne ablačné formy v Dobšinskej ľadovej jaskyni. In: *Aragonit*, ISSN 1335-213X, 2003, vol. 8, p. 3-7.

[2] BERBER, M., DARE, P. J., VANÍCEK, P., CRAYMER, M. R.: On the Application of Robustness Analysis to Geodetic Networks. *Proceedings of Canadian Society for Civil Engineering*, 31st Annual Conference, June 4-7, Moncton, NB, Canada, 2003.

[3] CASPARY, W.F.: Concepts of network and deformation analysis. First edition. Kensingthon: School of surveying The University of New South Wales, 1987, p.187. ISBN 0-85839-044-2.

[4] ZELIZŇAKOVÁ, V., GAŠINCOVÁ, S.: Analýza robustnosti polohovej geodetickej siete zriadenej v podzemných priestoroch Dobšinskej ľadovej jaskyne. In: Quaere 2013 : interdisciplinární konference doktorandů a odborných asistentů : mezinárodní vědecká konference : recenzovaný sborník příspěvků : vol. 3 : 20. - 24.května 2013, Hradec Králové. - Hradec Králové. Magnanimitas, 2013 p. 2835-2844, ISBN 978-80-905243-7-8

[5] IŽVOLTOVÁ, J. : Teória chýb a vyrovnávací počet I. Príklady ku cvičeniam. Žilinská univerzita, 2004, s. 121,. ISBN 9788080702397.

[6] VANÍČEK, P., CRAYMER, M. R., KRAKIWSKY, E. J.: Robustness analysis of geodetic horizontal networks. In: *Journal of Geodesy*, 2001, vol. 75, p. 199 – 209.

[7] WEISS, G., ŠÜTTI, J.: Geodetické lokálne siete I. 1.vydanie. Košice, Vydavateľstvo Štroffek, 1997, s.130. ISBN 80-967636-2-8 .

[8] FABIÁN, M., KOŽÁR, J., SOKOL, Š.: Varianty odhadu parametrov 1. a 2. rádu v geodetickej sieti budovanej po etapách. Geodetický a kartografický obzor, 37 (79), 1991, č. 5, s. 89-93.

[9] VYHLÁŠKA č. 87/2013 Úradu geodézie, kartografie a katastra Slovenskej republiky z 8. apríla 2013, ktorou sa mení a dopĺňa vyhláška Úradu geodézie, kartografie a katastra Slovenskej republiky č. 461/2009 Z. z., ktorou sa vykonáva zákon Národnej rady Slovenskej republiky č. 162/1995 Z. z. o katastri nehnuteľností a o zápise vlastníckych a iných práv k nehnuteľnostiam (katastrálny zákon) v znení neskorších predpisov v znení vyhlášky Úradu geodézie, kartografie a katastra Slovenskej republiky č. 74/2011 Z. z.

[10] www.ssj.sk/jaskyne/spristupnene/dobsinska-ladova

RESUMÉ

Vytvorená meračská sieť po svojom dobudovaní aj v spodnej časti Dobšinskej ľadovej jaskyne umožní vytvorenie jej presného digitálneho modelu, umožňujúceho na základe opakovaných expedičných meraní popísať exaktnými aj empirickými závislosťami fyzikálne procesy zmien jej ľadovej výplne. Kvantitatívne nové údaje o sezónnych, cyklických a trendových zmenách ľadovej výplne v Dobšinskej ľadovej jaskyni vo vzťahu k zmenám a sezónnemu režimu klimatických procesov, doplnia doterajšie poznatky o tejto významnej, z celosvetového hľadiska unikátnej zaľadnenej jaskyne pre jej bezpečné a trvalo udržateľné využívanie.