TROJ-VARIANTNÉ SPRACOVANIE A VYROVNANIE OBSERVÁCIÍ A URČOVANÝCH ODHADOV

THREE-VARIANT PROCESSING AND ADJUSTMENT OF OBSERVATIONS AND DETERMINED ESTIMATIONS

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Abstrakt

Predmetný článok sa zaoberá trivariántnym spracovaním a určením odhadov neznámych parametrov v geodetickej sieti observovanej technológiu využívajúcou signály globálnych navigačných satelitných systémov v rokoch 2004, 2008 a 2011. Cieľom práce je zhodnotiť vplyv použitej metódy vyrovnania na odhad parametrov prvého a druhého rádu geodetickej siete a prezentovať výsledky analýzy pretvorení s grafickou vizualizáciu jednotlivých spracovaní a analýz. Na spracovanie a vyrovnanie observácií boli použité MNŠ a robustné M-odhady podľa Hubera a Hampela. Pri analýze stability bodov poukázali všetky 3 metódy spracovania na posun bodu 5005 v epoche 04-08, čo potvrdzujú aj grafické vizualizácie pomocou konfidenčných elipsoidov chýb.

Abstract

The present article deals with three-variant processing and finding estimates of unknown parameters in a geodetic network by the technology of global navigation satellite systems in 2004, 2008 and 2011. The assessment of the impact of a used method of adjustment on the estimation of parameters of the first and second order of the geodetic network and the presentation of results of a deformation analysis with graphical visualisation of individual processing and analyses are the objectives of this paper. An LSM method and robust M-estimates according to Huber and Hampel were used for the processing and adjustment of observations. All three processing methods showed a displacement of point No. 5005 in the epoch 04-08 in the analysis of the stability of points, which is also confirmed by graphical visualisations using confidence error ellipsoids.

Keywords: geodetic network, GNSS observations, LMS, robust M-estimate, error ellipsoid

1 INTRODUCTION

Monitoring the stability of dynamically loaded water constructions is elaborated in the "Technical and Safety Supervision" (TaSS) approved by the Ministry of Environment of the Slovak Republic according to [19]. Details about the safety of water constructions are specified in Act No 364/2004 Coll., according to [18]. The spatial changes of water construction objects are surveyed with geodetic methods within the technical and safety supervision of water constructions.

Currently, the TaSS is realized over almost sixty water structures of regional importance in Slovakia. Systematic monitoring, consisting of collecting, processing, assessment and archiving of measured data at regular intervals, is required to obtain objective information and, if necessary, for early warning of instability of a water structure. Only terrestrial methods using direct lines of sight between individual points were known in the establishment of a geodetic network and a water structure. The topic is actual with respect to the determination of parameters of a geodetic network as currently there is only and exclusive use of GNSS technology in some cases of survey controls by reason of the disappearance of mutual lines of sight due to the growth of vegetation, for example in national parks.

The aim of this paper is to point out the possibilities of processing and analyzing variables obtained in a geodetic network by the GNSS technology. To better understand the behaviour of the area of interest beneath the upper reservoir of the pumped storage hydro power plant (PSHPP) Čierny Váh, a deformation analysis of this area was realized.

2 ADJUSTMENT METHODS IN SURVEYING

Methods of adjustment are based on the minimum condition of a norm of the vector of corrections. The norm is a number assigned to each n-dimensional vector that characterizes its size. In geodesy, objective functions of the following types are the most commonly used according to [1], [2]:

$$\rho(v_i)=\left(\sum_{i=1}^{n}|v_i|^{p}\right)^{\frac{1}{p}}=\min\,,\quad i\in\langle 1,n\rangle \tag{1}$$

The p parameter specifies a special type of an objective function. The parameter *p=2 - Least Squares Method* (LSM) *(L2-norm)* is most commonly used and is expressed by the objective function in the following form:

$$\rho(v_i)=\left(\sum_{i=1}^{n}v_i^{2}\right)^{\frac{1}{2}}=\min \tag{2}$$

The LSM provides an unbiased and the best estimate only for a normal distribution of errors in the set of measured variables. If the measured variables are weighted by systematic errors and mistakes (yawing values), the LSM is still effectively usable. This method also has the feature that larger errors of variables tries to decompose into smaller parts, thereby unacceptably distorting estimates of the adjustment procedure. Identifying and locating mistakes and systematic errors weighting several measured variables that would be either cleaned or excluded from the files entering the adjustment procedure is the objective of the reliable processing of measured variables prior to their evaluation according to [14], [16]. The majority of robust adjustments used in geodesy modify the existing LSM to make it robust. When using the robust LSM, the weight of measurement changes in each iteration using a weight function. When using the robust method for estimation, the minimised function $v^T v$ is replaced with the so-called loss function according to [3], [4], [7]: $\rho(v_i)=\min$ which generates the influence function $\psi(v_i)$ characterising the influence of errors on adjusted values:

$$\sum_{i=1}^{n}\psi(v_i)=\sum_{i=1}^{n}\frac{\partial\rho(v_i)}{\partial v_i}=0\,. \tag{3}$$

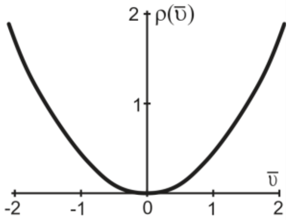
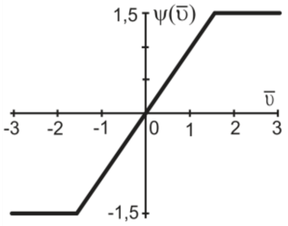
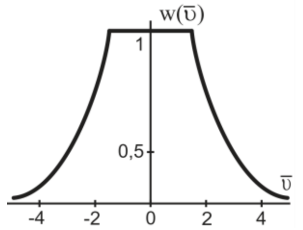
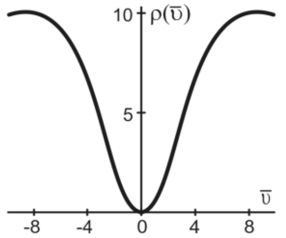
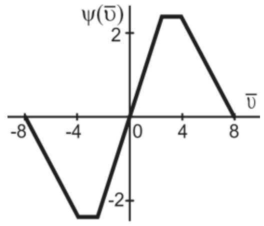
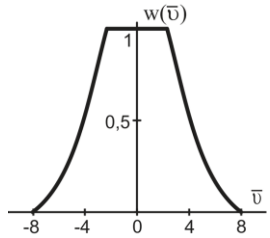
In order that the adjustment will have the nature of a robust estimate, it is suitable to carry out it using the iteration method with variable weighing, i.e. so that the weight p_i of observation l_{ij} was determined in each iteration step as a corrective function (weight function):

$$p(v_i)=\frac{\psi(v_i)}{v_i}\,. \tag{4}$$

According to [9], [10], [12], Huber's robust M-estimate, Hampel's robust M-estimate and Beweight's robust M-estimate are the most used estimates. The functions of selected estimates are shown in

Tab. 1.

Tab. 1 Function of Huber's and Hampel's robust M-estimates.

<table>
<tr><td></td><td>Loss function
$\rho(v_i)=$</td><td>Influence function
$\psi(v_i)=\frac{\partial\rho(v_i)}{\partial v_i}=$</td><td>Corrective function
$p(v_i)=\frac{\psi(v_i)}{|v_i|}=$</td></tr>
<tr><td rowspan="2">Huber's robust M-estimate</td><td>$\begin{cases}\frac{1}{2}v_i^2 & |v_i|<k\\ k|v_i|-\frac{1}{2}k^2 & |v_i|\geq k\end{cases}$</td><td>$\begin{cases}v_i & |v_i|<k\\ k\ sign(v_i) & |v_i|\geq k\end{cases}$ Damping constant: k=1,5</td><td>$\begin{cases}1 & |v_i|<k\\ \frac{k}{|v_i|} & |v_i|\geq k\end{cases}$</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td rowspan="2">Hampel's robust M-estimate</td><td>$\begin{cases}\frac{1}{2}v_i^2 & |v_i|<a\\ a|v_i|-\frac{1}{2}a^2 & a\leq|v_i|<b\\ \frac{a}{2}\left(b-a+c-\frac{(c-|v_i|)^2}{c-b}\right) & b\leq|v_i|<c\\ \frac{a}{2}(b-a+c) & c\leq|v_i|\end{cases}$</td><td>$\begin{cases}v_i & |v_i|<a\\ a\ sing(v_i) & a\leq|v_i|<b\\ a\frac{c-|v_i|}{c-b}sing(v_i) & b\leq|v_i|<c\\ 0 & c\leq|v_i|\end{cases}$
Damping constants:
a=2, b=4, c=8;</td><td>$\begin{cases}1 & |v_i|<a\\ \frac{a}{|v_i|} & a\leq|v_i|<b\\ a\frac{c-|v_i|}{(c-b)|v_i|} & b\leq|v_i|<c\\ 0 & c\leq|v_i|\end{cases}$</td></tr>
<tr><td>
</td><td>
</td><td>
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3 PROCESSING GEODETIC NETWORK

Likewise terrestrial measurements and their results are weighted by errors, so the GNSS observations and their results are affected by different factors that decrease their accuracy. Therefore, the adjustment process is also applied to the GNSS observations in order to determine the best estimations of determined parameters. The processing and adjustment of a network (with full or incomplete rank) has common input variables, but with different structures and content of relevant variables. A Gauss Markov model (GMM) is the most commonly used method for adjustment of a general geodetic network, defined as follows according to [5], [8], [12], [13], [15]:

$$\begin{aligned}&\mathbf{v}=\mathbf{A}\mathbf{d}\hat{\mathbf{C}}-\mathbf{dL}=\mathbf{A}(\hat{\mathbf{C}}-\mathbf{C}^{\circ})-(\mathbf{L}-\mathbf{L}^{\circ}), && \text{-functional part,}\\ &\mathbf{\Sigma}_{\mathbf{L}}=s_0^2\mathbf{Q}_{\mathbf{L}}, && \text{-stochastic part,}\end{aligned} \qquad (5)$$

where:

v – vector of corrections of observed values,

$\mathbf{d\hat{C}}$ – vector of complements of adjusted values of determining coordinates,

dL – vector of auxiliary observations,

A – design matrix (of a partial derivations),

$\mathbf{\Sigma_L}$ – covariance matrix of observations,

s_0^2 – unit a posteriori variance of the realised estimation process,

$\mathbf{Q_L}$ – cofactor matrix of observations.

The adjustment procedure consists of the following steps:

1 arrangement of the input data,

2 definition of model equations,

3 auxiliary calculations,

4 creation of a configuration matrix,

5 calculation of estimations,

6 expression of the accuracy of a net.

4 GNSS OBSERVATIONS OF NETWORK OF PSHPP ČIERNY VÁH

As terrestrial or GNSS observations have different advantages and disadvantages, selecting the technology to be used can be done based on their understanding. The GNSS technology, in contrast to terrestrial technologies, is not dependent on the direct visibility between points. Therefore, it is not necessary to make forest paths through bushy or forest stands in the areas of increased protection of nature. The requirement of unshielded sky or direct visibility to satellites must be met for GNSS observations.

Seven points of the geodetic network are located around the crest of the upper reservoir of the hydro power plant Čierny Váh (Fig. 1). The points are monumented by heavy monumentation around the reservoir, labelled numerically in the range of 5001 – 5007. The monumentation of the points was performed after the construction of the hydro power plant. The observations were realized in three independent epochs (Tab. 2) for the purpose of the deformation monitoring of stability or instability of the observed points of the geodetic network. The measurement was realized using a static method successively over all network points: 5001 to 5007 in all three epochs. The time of signal receiving was set from 40 minutes to 7 hours (for reference points). 11 GPS/GNSS vectors (in the form of a 7-gon) for each epoch of observations resulted from the processing of observations (Fig. 1).

Tab. 2 Realized observations of the geodetic network

Epoch	Month/year	Observation days	Number and kind of reciever
04	April 2004	2	2 x GPS Sokkia Stratus
08	July 2008	1	4 x GPS Sokkia Stratus
11	October 2011	1	3 x GNSS Leica GPS 1200/900CS

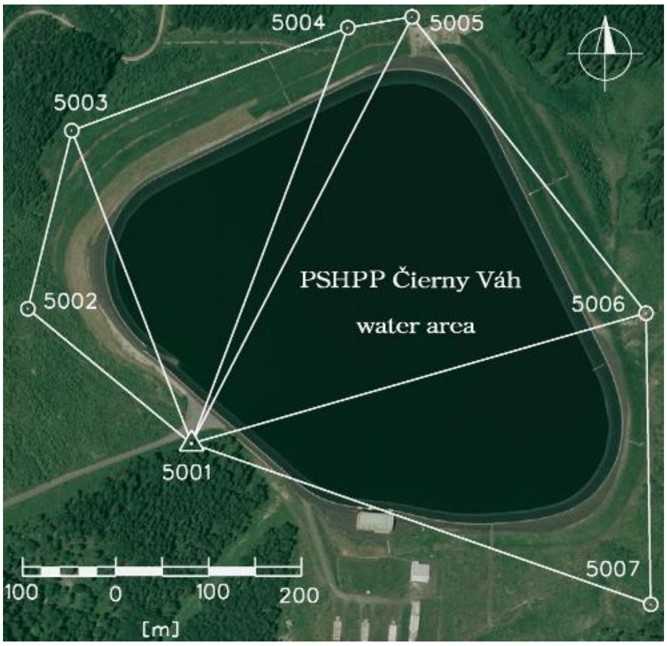

Fig. 1 The network structure around the crest of the upper reservoir of the PSHPP Čierny Váh

5 PROCESSING OBSERVATIONS

Observations were processed in the Spectrum Survey and Leica Geo Office software. Spatial orthogonal coordinates *X*, *Y*, *Z* and their coordinate differences were the results of processing using the software. The data of GPS and GNSS vectors were subsequently processed based on the Gauss-Markov estimation model (GMM – adjustment of indirect measurements) as a GMM with full rank.

The point 5001 was selected as a reference point of the geodetic network and the coordinates of the point 5001 were processed by post processing together with the files in the RINEX 2.11 format from the reference station SKLM according to [11]. The adjusted coordinates of the network points $\hat{C}$ are determined by variables measured in the network and pre-processed which in this case are represented by GNSS observation vectors arranged to the vector L with the following structure:

$$L=\begin{pmatrix} {}^{04}L \\ {}^{08}L \\ {}^{11}L \end{pmatrix}, \text{ where } {}^{t}L=\left({}^{t}\Delta XYZ_{ij}\right), \text{ while } t=04,08,11 \text{ and } \Delta XYZ_{ij}=\begin{pmatrix} \Delta X_{ij} \\ \Delta Y_{ij} \\ \Delta Z_{ij} \end{pmatrix}. \tag{6}$$

The observation vector L consist of $3\ x\ 11$ spatial vectors of observations ΔXYZ_{ij}, i.e. $3\ x\ 33$ of observation components $\Delta X_{ij}, \Delta Y_{ij}, \Delta Z_{ij}$. For the solution of processing the deformation network observed in three epochs, the three-variant processing and adjustment of observations and determined estimations of the adjusted coordinates were used, since there is no statistically significant difference between the outputs from a separate or common adjustment. For the three-variant solution, the GMM is defined as:

$$\underset{(99,1)}{\begin{pmatrix} {}^{04}v \\ {}^{08}v \\ {}^{11}v \end{pmatrix}}=\underset{(99,54)}{\begin{pmatrix} {}^{04}A & & 0 \\ & {}^{08}A & \\ 0 & & {}^{11}A \end{pmatrix}}\cdot\underset{(54,1)}{\begin{pmatrix} {}^{04}d\hat{C} \\ {}^{08}d\hat{C} \\ {}^{11}d\hat{C} \end{pmatrix}}-\underset{(99,1)}{\begin{pmatrix} {}^{04}dL \\ {}^{08}dL \\ {}^{11}dL \end{pmatrix}}, \tag{7}$$

where ${}^{04}A={}^{08}A={}^{11}A=A$ is the matrix of partial derivations **L°** according to **C°**.

Considering the number of processed epochs (1, 2 or all 3), no differences between adjustments were identified, therefore it is the discretion of the user performing the processing, which of the specified methods will be chosen. Especially, a simple addition of more epochs into the common processing is an advantage of the software solution, which subsequently provides common outputs for all processed epochs in one output file in text format, for example *.txt. For the processing in the Matlab software, it is necessary to write an algorithm for processing the values obtained by observations, where it is also possible to select individual calculation steps that should be displayed in the output file from the processing, or create graphical visualisation of the determined unknown parameters (resulting values).

6 ESTIMATES OF ADJUSTED COORDINATES

The coordinate values of determined points of the network $\hat{C}$ are dependent on the used estimation method of unknown parameters (LSM, robust M-estimation according to Huber or Hampel). The difference of the estimates of complements of the adjusted coordinates $d\hat{C}$ is dependent on the used estimation method, due to assigning cofactors (weights) of varying size based on the size of corrections for individual algorithms of robust M-estimations. The Estimates of the adjusted coordinates for the LSM, and the robust M-estimations according to Huber and Hampel are presented in Tab. 3.

Tab. 3 Coordinates from adjustments by the LSM and according to Huber and Hampel

$^{\tau}t$	Point	LSM XETRS89 [m]	LSM YETRS89 [m]	LSM ZETRS89 [m]	Huber XETRS89 [m]	Huber YETRS89 [m]	Huber ZETRS89 [m]	Hampel XETRS89 [m]	Hampel YETRS89 [m]	Hampel ZETRS89 [m]
^{04}t	5002	3 941 063.358	1 427 021.983	4 792 984.567	3 941 063.358	1 427 021.984	4 792 984.567	3 941 063.358	1 427 021.984	4 792 984.567
	5003	0 896.381	6 998.783	3 089.955	0 896.381	6 998.783	3 089.955	0 896.382	6 998.783	3 089.955
	5004	0 722.170	7 243.221	3 194.259	0 722.170	7 243.220	3 194.259	0 722.171	7 243.220	3 194.259
	5005	0 690.589	7 304.206	3 208.553	0 690.590	7 304.206	3 208.552	0 690.590	7 304.206	3 208.552
	5006	0 816.179	7 638.525	3 016.659	0 816.180	7 638.524	3 016.659	0 816.180	7 638.524	3 016.659
	5007	1 027.267	7 741.653	2 811.096	1 027.267	7 741.652	2 811.096	1 027.267	7 741.652	2 811.096
^{08}t	5002	3 941 063.360	1 427 021.986	4 792 984.571	3 941 063.360	1 427 021.985	4 792 984.571	3 941 063.360	1 427 021.985	4 792 984.571
	5003	0 896.375	6 998.784	3 089.950	0 896.375	6 998.783	3 089.951	0 896.375	6 998.783	3 089.950
	5004	0 722.168	7 243.221	3 194.260	0 722.167	7 243.223	3 194.260	0 722.167	7 243.223	3 194.260
	5005	0 690.584	7 304.226	3 208.559	0 690.584	7 304.226	3 208.560	0 690.584	7 304.227	3 208.560
	5006	0 816.169	7 638.519	3 016.661	0 816.169	7 638.517	3 016.663	0 816.169	7 638.518	3 016.662
	5007	1 027.259	7 741.649	2 811.098	1 027.259	7 741.648	2 811.099	1 027.259	7 741.649	2 811.099
^{11}t	5002	3 941 063.364	1 427 021.989	4 792 984.566	3 941 063.364	1 427 021.990	4 792 984.566	3 941 063.364	1 427 021.990	4 792 984.566
	5003	0 896.378	6 998.786	3 089.953	0 896.378	6 998.788	3 089.953	0 896.377	6 998.788	3 089.953
	5004	0 722.170	7 243.214	3 194.264	0 722.170	7 243.214	3 194.265	0 722.170	7 243.214	3 194.265
	5005	0 690.579	7 304.220	3 208.561	0 690.578	7 304.220	3 208.563	0 690.578	7 304.220	3 208.563
	5006	0 816.174	7 638.516	3 016.657	0 816.174	7 638.516	3 016.659	0 816.174	7 638.516	3 016.660
	5007	1 027.261	7 741.644	2 811.092	1 027.261	7 741.644	2 811.093	1 027.261	7 741.644	2 811.094

Tab. 4 Errors from adjustments by the LSM and according to Huber and Hampel

	Point	LSM Epoch 04	LSM Epoch 08	LSM Epoch 11	Huber Epoch 04	Huber Epoch 08	Huber Epoch 11	Hampel Epoch 04	Hampel Epoch 08	Hampel Epoch 11
Mean coordinate errors spi [mm]	5002	5.22	5.32	5.22	4.48	4.67	4.52	4.60	4.57	4.74
	5003	4.61	4.70	4.61	4.20	3.31	3.67	4.26	3.34	3.99
	5004	4.51	4.60	4.51	4.16	3.60	4.14	4.29	3.66	4.49
	5005	4.54	4.62	4.54	4.03	3.29	3.96	4.15	3.26	4.12
	5006	4.67	4.75	4.67	4.05	3.78	4.31	4.16	3.59	4.55
	5007	5.32	5.42	5.32	4.16	3.70	4.53	4.36	3.67	4.64
		Average coordinate errors sp [mm]								
		4.840			4.030			4.136		
Mean spatial errors sXYZi [mm]	5002	3.01	3.07	3.01	2.58	2.70	2.61	2.66	2.64	2.74
	5003	2.66	2.71	2.66	2.42	1.91	2.12	2.46	1.93	2.30
	5004	2.61	2.65	2.61	2.40	2.08	2.39	2.48	2.11	2.59
	5005	2.62	2.67	2.62	2.33	1.90	2.29	2.40	1.88	2.38
	5006	2.69	2.74	2.69	2.34	2.18	2.49	2.40	2.07	2.63
	5007	3.07	3.13	3.07	2.40	2.14	2.62	2.52	2.12	2.68
		Average spatial errors sXYZ [mm]								
		2.794			2.327			2.388		

Standard deviations (variances) of estimates of the adjusted coordinates $s_{\hat{C}}$ are also dependent on the selection of the adjustment method and related creation of a cofactor matrix Q_L. The comparison of mean coordinate errors $s_{\hat{X}\hat{Y}\hat{Z}_i}$, average coordinate errors $s_{\hat{X}\hat{Y}\hat{Z}}$, mean spatial errors s_{p_i} and average spatial errors s_p by using different methods of adjustment are presented in Tab. 4. The lowest average coordinate error was obtained by the robust M-estimation according to Huber $\bar{s}_{\hat{C}}=2.327mm$, then by the M-estimation according to Hampel $\bar{s}_{\hat{C}}=2.388mm$ and the highest average coordinate error was calculated by the LSM method $\bar{s}_{\hat{C}}=2.794mm$.

In addition to the numerical determination of point positions in space, subsequent visualisation using the Matlab 7.12.0 software was done with the accuracy of their determination by absolute confidence ellipsoids; their representation is shown in Fig. 2. This visualisation presents the representation of all epochs (^{04}t, ^{08}t and ^{11}t) of adjustment according to the used estimation methods. The coordinates on the axis X-ETRS-89, Y-ETRS-89, Z-ETRS-89 are in meters.

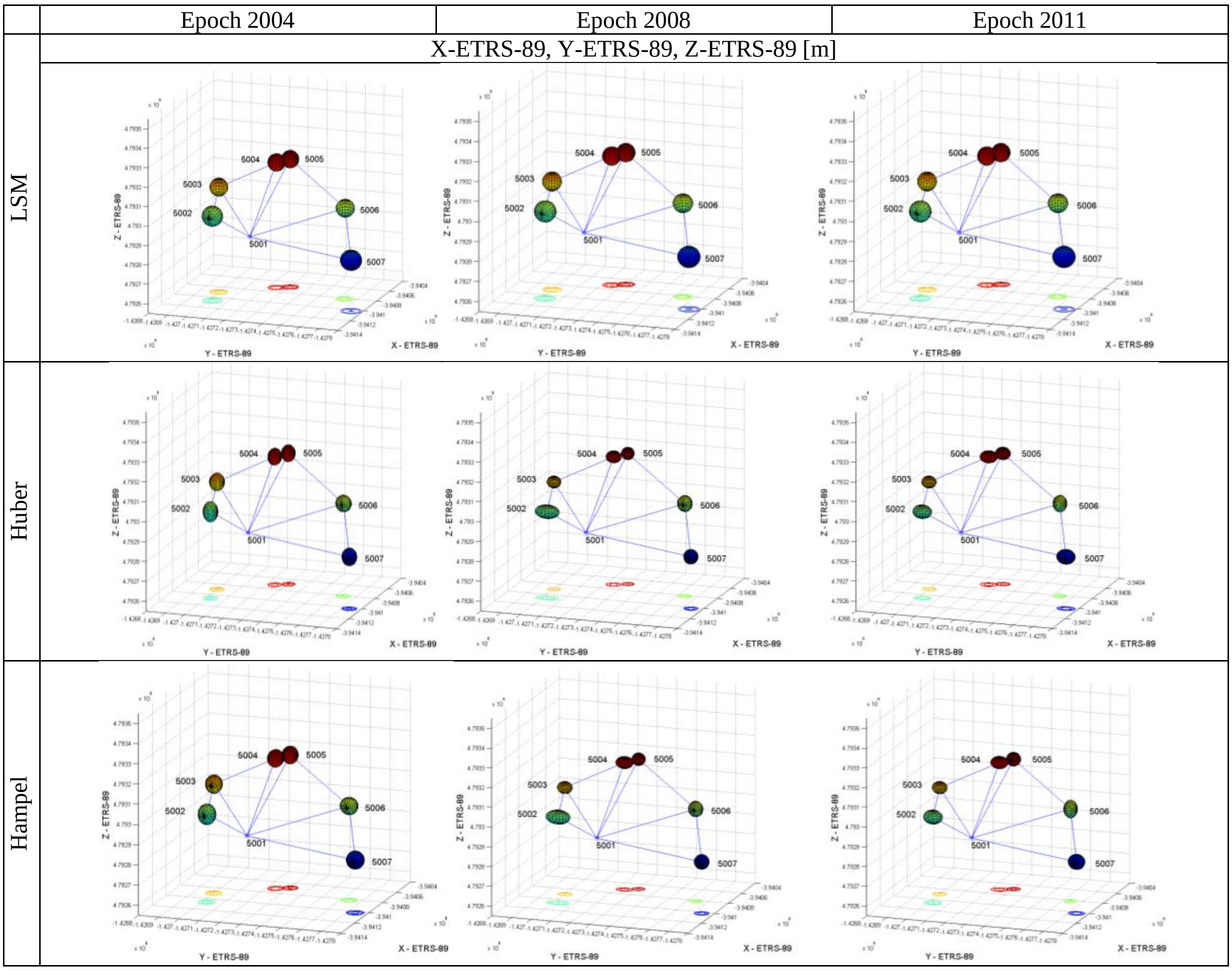

Fig. 2 Three-variant adjustment of the network in the epochs 2004, 2008, 2011

7 ASSESSMENT OF POINT STABILITY

The stability examination of points is done by a mutual comparison of coordinate changes between the previous t and the following epoch $t+1$. Their differences in directions of individual coordinate axes are determined from the estimations of the adjusted coordinates according to [13], [15]:

$$^{t,t+1}\Delta\hat{C}_i = {}^{t+1}\hat{C}_i - {}^{t}\hat{C}_i = \begin{pmatrix} ^{t,t+1}\Delta\hat{X}_i \\ ^{t,t+1}\Delta\hat{Y}_i \\ ^{t,t+1}\Delta\hat{Z}_i \end{pmatrix} = \begin{pmatrix} ^{t+1}\hat{X}_i - {}^{t}\hat{X}_i \\ ^{t+1}\hat{Y}_i - {}^{t}\hat{Y}_i \\ ^{t+1}\hat{Z}_i - {}^{t}\hat{Z}_i \end{pmatrix}, \tag{8}$$

that represent the size of a position change of the i-th point between epochs t and $t+1$ in the direction of the axes X, Y and Z, thus 1-dimensional or axial displacements. The size of the spatial point position change as a spatial displacement is determined by the equation:

$$^{t,t+1}\Delta\hat{X}\hat{Y}\hat{Z} = \sqrt{^{t,t+1}\Delta\hat{X}^2 + {}^{t,t+1}\Delta\hat{Y}^2 + {}^{t,t+1}\Delta\hat{Z}^2}\,. \tag{9}$$

The geometrical structures for identical (unchanged) positions of points should be stochastically identical – congruent. It is necessary to verify whether the identified changes indicate a real movement of an object point between epochs, or it is just a displacement arising out of the propagation of measurement errors by statistical testing where a stochastic coordinate identity is assessed on the basis of a certain probability of normal distribution.

The significant stability or instability of the deformation network is examined using the *global congruence test*, for which the null hypothesis is expressed in the form according to [13], [15], [17]:

$$H_0:\ ^{t,t+1}\Delta\hat{C}_i = {}^{t+1}\hat{C}_i - {}^{t}\hat{C}_i = 0\,, \tag{10}$$

expressing the assumption that the observed network remains stable. The acceptance or rejection of H_0 results from the decision-making process by comparing the critical value F_{crit} with the testing criterion of the relevant statistical test:

$$T = \frac{\dfrac{\Delta\hat{C}^T \cdot ({}^{j}Q_{\hat{C}} + {}^{j+1}Q_{\hat{C}} - 2^{j,j+1}Q_{\hat{C}}) \cdot \Delta\hat{C}}{\dfrac{{}^{j}(v^T Q_L^{-1} v) + {}^{j+1}(v^T Q_L^{-1} v)}{{}^{j}(n-k) + {}^{j+1}(n-k)}}, }{f_1} \approx F_{krit} = F_{\alpha}(1-\alpha, f = k_i, f' = n-k) \quad (11)$$

If $T<F_{krit}$, all observed points in $\Delta t={}^{j+1}t-{}^{j}t$ are considered stable, i.e. their position did not change, because there was no influence of deformation forces. The network is congruent.

If $T>F_{krit}$, some of the points have significantly changed their position for the period $\Delta t={}^{j+1}t-{}^{j}t$, due to the effects of deformation forces.

The determination of the points, at which the displacement occurred in the period Δt, is performed by an *identification test of congruency*. For the localization of unstable points, a numerical value R (applies to k points) is decomposed into its partial components R_i, $i=1,2,\ldots k$, related to individual points of the net. The decomposition of R can be done by an approximating procedure, i.e. its proportion in R_i will be determined for each point by the value according to [13], [15], [17]:

$$R_i=\Delta\hat{C}_i^T \cdot Q_{\Delta\hat{C}_i}^{-1} \cdot \Delta\hat{C}_i , \quad (12)$$

where $Q_{\Delta\hat{C}_i}^{-1}$ are elements only from the main diagonal of the matrix $Q_{\Delta\hat{C}}^{-1}={}^{j}Q_{\hat{C}}+{}^{j+1}Q_{\hat{C}}-2^{j,j+1}Q_{\hat{C}}$.

The localization test statistic T_i for individual points with the critical value is as follows:

$$T_i=\frac{R}{k_i \bar{s}_0^2} \; (i=1,2,\ldots k_i) \approx F_{krit}=F_{\alpha}(1-\alpha, f=k_1, f'=n-k) \quad (13)$$

If $T_i \le F_{krit}$, the point can be considered as stable for a given period and the changes of the point coordinates in ${}^{j+1}t$ against their values in ${}^{j}t$ are not significant.

If $T_i>F_{krit}$, it is expresses the instability of the point and a spatial change of the relevant point can be accepted at a level of significance α as a result of deformation forces.

8 RESULTS OF DEFORMATION ANALYSIS

Two pairs of epochs were created from three observation epochs for the deformation analysis of stability of the monitored geodetic control. The pair of first and last epoch was created, i.e. ${}^{04}t$, ${}^{11}t$ and the pair of penultimate and last epoch, i.e. ${}^{08}t$, ${}^{11}t$. For both pairs of epochs, deformation vectors with differences between estimates of coordinates were created according to (9) for all 3 methods applied in the process of estimation of the adjusted determined parameters. The differences between the estimates of the adjusted coordinates were expressed numerically in the direction of individual axes $\Delta\hat{X}, \Delta\hat{Y}, \Delta\hat{Z}$ for epochs ${}^{04}t$, ${}^{11}t$ and epochs ${}^{08}t$, ${}^{11}t$ (Tab. 5). For epochs ${}^{04}t$, ${}^{11}t$ and epochs ${}^{08}t$, ${}^{11}t$ (Tab. 6), also spatial differences $\Delta\hat{X}\hat{Y}\hat{Z}$ were calculated.

The testing showed a statistically significant difference between coordinates of the same point, namely the point 5005, in the direction of axes *X*, *Y* and *Z* for all methods used for processing in the epoch ${}^{04}t$ - ${}^{11}t$ (Tab. 5 – highlighted in colour). By the statistical testing, the spatial displacement of the point 5005 (Tab. 6 – highlighted in colour) shows the largest change by using the robust M-estimation according to Hampel in the epoch ${}^{04}t$ - ${}^{11}t$, namely 20.70 mm. The use of the LSM method showed the smallest change in this epoch, namely 18.73 mm. The robust M-estimation according to Huber estimated the spatial change to 20.57 mm. Because of using different methods, the differences in the size of displacements between individual estimations vary from each other up to about 2 mm (between the LSM method and the M-estimations).

Tab. 5 Estimation differences of adjusted coordinates in the direction of axes

Point		epoch 04t and 11t						epoch 08t and 11t					
		LSM		Huber		Hampel		LSM		Huber		Hampel	
		Shift [mm]	T	Shift [mm]	T	Shift [mm]	T	Shift [mm]	T	Shift [mm]	T	Shift [mm]	T
5002	ΔX	6.23	1.44	6.17	1.83	5.88	1.91	3.74	0.75	3.67	1.06	3.63	1.04
	ΔY	5.58	1.12	6.01	1.13	6.29	1.57	3.24	0.55	4.48	0.63	4.74	0.75
	ΔZ	-1.26	0.06	-0.92	0.03	-0.92	0.04	-5.35	1.53	-5.33	2.27	-5.32	2.26
5003	ΔX	-3.68	0.63	-3.75	0.73	-4.25	1.16	2.46	0.41	2.34	0.52	2.26	0.50
	ΔY	3.41	0.54	4.25	0.83	4.80	1.26	2.39	0.39	4.79	1.69	4.67	1.56
	ΔZ	-2.56	0.31	-1.88	0.17	-1.88	0.20	2.28	0.36	2.34	0.57	2.37	0.58
5004	ΔX	0.67	0.02	-0.49	0.01	-1.12	0.06	2.65	0.48	2.35	0.36	2.14	0.35
	ΔY	-6.67	2.26	-6.35	1.73	-6.66	2.16	-6.99	3.59	-9.03	4.52	-9.15	4.38
	ΔZ	4.48	0.98	5.13	1.31	5.25	1.69	4.17	1.23	4.35	2.02	4.42	2.07
5005	ΔX	-10.22	4.96	-11.62	6.79	-11.73	8.42	-5.37	1.98	-5.61	2.80	-5.72	3.03
	ΔY	13.23	8.72	13.41	8.94	13.30	10.83	-6.22	2.79	-6.26	2.96	-6.93	4.01
	ΔZ	8.46	3.48	10.41	4.53	10.68	6.13	2.20	0.34	2.70	0.69	2.90	0.74
5006	ΔX	-4.97	1.14	-5.22	1.40	-5.42	1.84	5.35	1.91	5.98	3.03	5.39	2.59
	ΔY	-8.65	3.31	-7.75	2.85	-7.89	3.59	-2.63	0.44	-1.38	0.14	-2.10	0.36
	ΔZ	-1.45	0.10	0.10	0.00	1.42	0.09	-3.54	0.86	-2.11	0.24	-1.68	0.16
5007	ΔX	-5.96	1.30	-6.11	1.88	-6.21	2.12	1.22	0.08	1.49	0.17	1.20	0.11
	ΔY	-8.65	2.49	-8.12	2.38	-8.17	3.06	-5.18	1.29	-4.42	1.04	-4.81	1.47
	ΔZ	-3.73	0.51	-2.97	0.32	-2.33	0.26	-5.75	1.75	-5.06	1.73	-4.84	1.60
Critical value:		T_{crit} = 4.043											

Tab. 6 Estimation differences of adjusted coordinates in space

Point		epoch ^{04}t and ^{11}t						epoch ^{08}t and ^{11}t					
		LSM		Huber		Hampel		LSM		Huber		Hampel	
		Shift [m]	T	Shift [m]	T	Shift [m]	T	Shift [m]	T	Shift [m]	T	Shift [m]	T
5002	$\Delta\hat{X}\hat{Y}\hat{Z}$	8.46	0.87	8.66	0.99	8.66	1.17	7.29	0.94	7.87	1.32	7.99	1.35
5003		5.63	0.50	5.97	0.58	6.68	0.87	4.12	0.38	5.82	0.93	5.70	0.88
5004		8.06	1.09	8.18	1.02	8.55	1.30	8.56	1.77	10.29	2.30	10.39	2.27
5005		18.73	5.72	20.57	6.75	20.70	8.46	8.51	1.70	8.83	2.15	9.44	2.59
5006		10.08	1.52	9.34	1.42	9.67	1.84	6.93	1.07	6.49	1.14	6.02	1.04
5007		11.15	1.43	10.59	1.53	10.52	1.81	7.84	1.04	6.88	0.98	6.93	1.06
Critical value:		T_{crit} = 2.798											

Visualisation of individual network points represents a graphical method of examination of the change of point positions (accommodation of systematic and measurement errors or displacements). As stated in the testing of the deformation vector in individual axes and space that these are statistically significant differences between the coordinates of the point 5005, i.e. its displacement occurred, the the graphical testing by absolute confidence ellipsoids confirms the numerical results from the adjustment epochs ^{04}t and ^{11}t and from the epochs ^{08}t and ^{11}t (Fig. 3 – 1st and 2nd lines).

The coordinates *Xr*, *Yr*, *Zr* displayed in Fig. 3 are reduced showing only decimal places of coordinates in metres. The displayed absolute confidence ellipsoids represent 95% area of the point occurrence in the relevant epoch and can be used for graphical examination of deformations of individual point positions (accommodation of systematic and measurement errors or displacements). If the absolute confidence ellipsoids do not penetrate each other, then the statistically significant displacement occurred at that point; otherwise it is the accumulation of measurement errors. In the case of the relative confidence ellipsoids, if the connecting line of point positions between 2 epochs exceeds the ellipsoid surface, then a statistically significant change of the point position occurred and is declared as a point displacement. The differences between the positions of the point 5005 are also shown by means of relative confidence ellipsoids for individual estimation methods of unknown parameters and epochs ^{04}t - ^{11}t and ^{08}t - ^{11}t (Fig. 3 – 3th and 4th lines).

The systems WGS84 or ETRS89 display an area (for example the area of SR) only in a general position. The individual changes in point positions in these systems do not provide sufficient information and graphical visualisation of displacements in the horizontal and vertical direction. Therefore, it was necessary to transform the coordinates of points in the epochs ^{04}t, ^{08}t and ^{11}t from the ETRS89 system to the coordinate system Uniform Trigonometric Cadastral Network (S-UTCN) according to [6], [7].

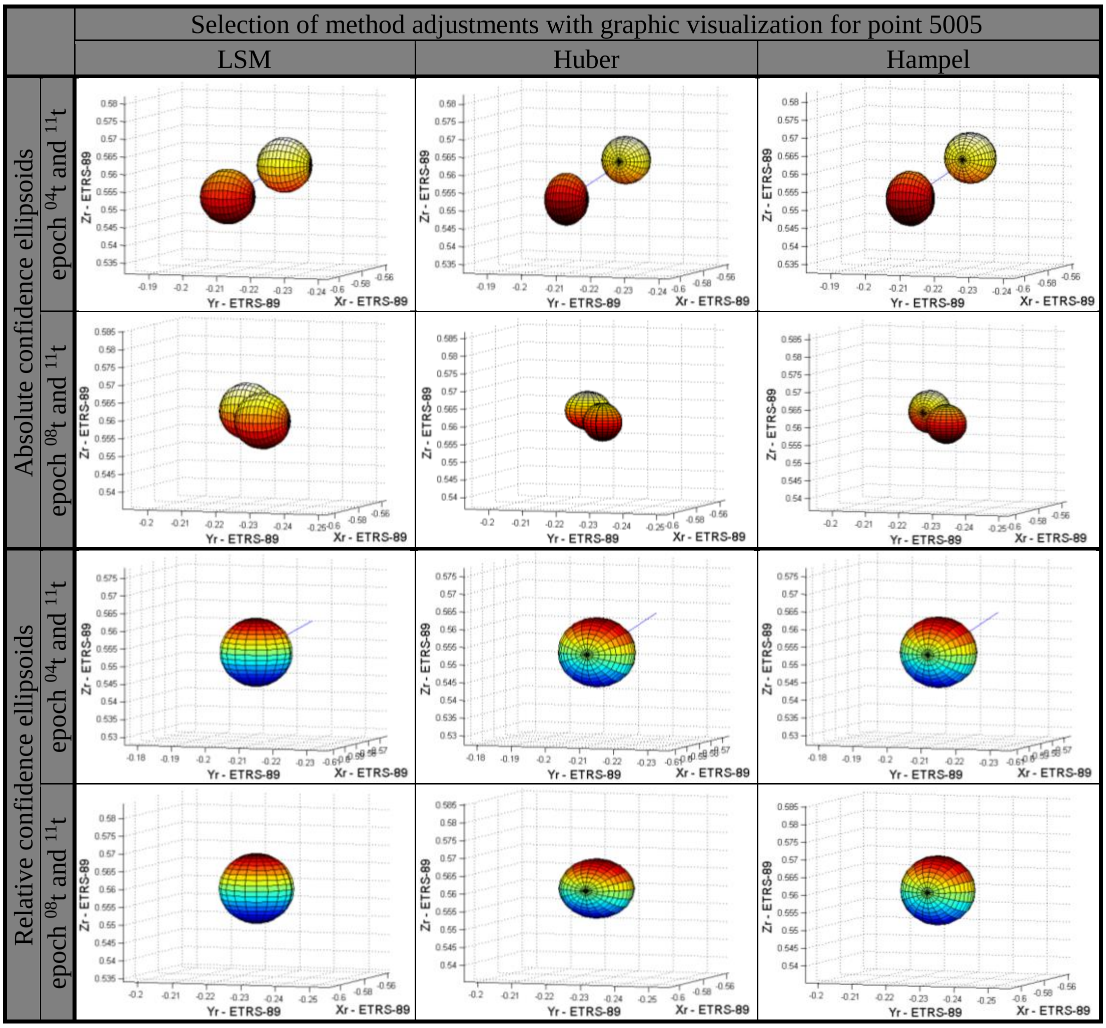

Fig. 3 Graphic visualisation of the point 5005 via confidence ellipsoids

9 CONCLUSION

The article provides the processing of the geodetic network of the upper reservoir of the pumped storage hydro power plant (PSHPP) Čierny Váh in the years 2004 – 2011. The analysis was realized based on the stage adjustment of GNSS vectors, by applying three selected methods of processing and adjustment of a geodetic network with the estimation of unknown parameters. The LSM method, robust M-estimation according to Huber, and robust M-estimation according to Humpel represented the selected methods of estimations. In the article, the estimations of parameters of the 1st and 2nd order of network structures and their statistical assessment in the area of deformation monitoring were solved.

The Gauss-Markov model of indirect measurements, solved as a GMM with full rank, was used for the processing. The epochs 2004, 2008 and 2011 were adjusted as a three-variant processing by using three methods for estimations of determined parameters. After the initial processing in the software supplied by distributors, the files of measured data were processed in the Matlab software with subsequent visualisation of the point positions with their accuracy in a form of ellipsoids.

However, a displacement of the point 5005 was demonstrated by all three methods. Other differences were identified as the result of the effect of systematic and measurement errors. Graphical testing using absolute and relative confidence ellipsoids that confirmed the results obtained by the processing were also realized. The use of Huber's and Hampel's robust M-estimates is an alternative to the application of an LSM method, which has a versatile use in practice in various areas of professional disciplines.

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RESUMÉ

Spracovanie geodetickej siete na prečerpávajúcej vodnej elektrárni PVE Čierny Váh pomocou technológie GNSS bolo vykonané za účelom určenia stability resp. nestability geodetickej siete umiestnenej na hornej korune hrádze. V článku bol zhodnotený vplyv použitej metódy vyrovnania na odhad parametrov prvého a druhého rádu sieťovej štruktúry a boli prezentované výsledky analýzy pretvorení s grafickou vizualizáciu jednotlivých metód spracovaní a analýz pretvorení geodetickej siete. Na trivariátne spracovanie a vyrovnanie observácií boli použité vybrané metódy MNŠ a robustné M-odhady podľa Hubera a Hampela. Analýza stability bodov poukázala u všetkých 3 metód spracovania na posun bodu 5005 v epoche 04-08, čo sa potvrdilo aj grafickou vizualizáciou využitím konfidenčných elipsoidov chýb.